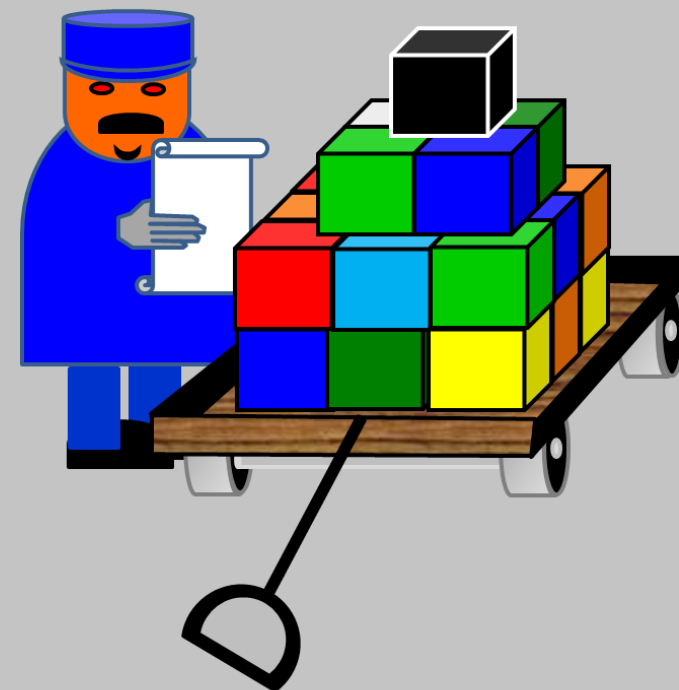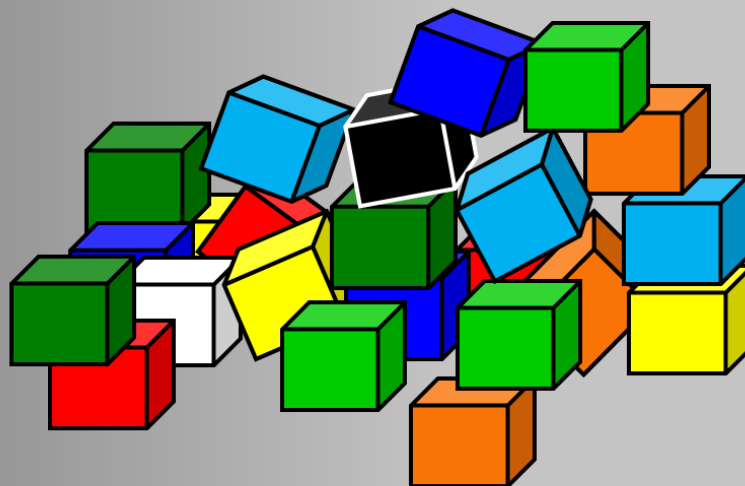# THEOREM OF THE DAY

Lagrange's Four-Squares Theorem *Any non-negative integer, n, may be written as a sum of four squares:*

$$n = w^2 + x^2 + y^2 + z^2,$$

where w, x, y and z are non-negative integers (some of which may be zero.)

The Mathematical Porter insists on piling boxes on his trolley in a pyramid of square layers to a height of at most four. Lagrange's theorem says this may be accomplished no matter how many boxes the porter has. Here, the pyramid illustrates $23 = 3^2 + 3^2 + 2^2 + 1^2$.

But now the porter finds he has overlooked a box! How will he restack 24 boxes on his trolley?

The result was known to Diophantus of Alexandria and was first explicitly asserted by Claude Gaspard Bachet de Méziriac, who translated Diophantus's *Arithmetica* into Latin in 1621. Its proof required a hundred and fifty years of work by modern mathematicians, culminating in Lagrange's complete proof of 1770. More generally, **Waring's Problem** (solved affirmatively but non-constructively in 1909 by Hilbert) asks if *any positive integer n can be written as a sum of at most a fixed number of k-th powers*. For instance, any non-negative integer can be written as a sum of 9 cubes; thus $23 = 2^3 + 2^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3$. Actually, this is one of only two numbers requiring 9 cubes (the other being 239) and it is still unknown whether, for large enough integers, 6 cubes might be enough (8042 is the largest integer known to require 7 cubes).
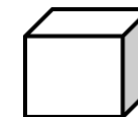
Web link: www.maths.lancs.ac.uk/~jameson/foursquares.pdf
Further reading: *Elementary Number Theory* by Gareth Jones and Mary Jones, Springer, Berlin, 1998.