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CHAPTER 2

GENERALITIES ABOUT PARTIAL *-ALGEBRAS

2.0 Introduction:

A brief account of partial *-algebras will be given in this section. Partial *-algebras are generalizations of *-algebras in which the usual requirement of closure under multiplication is waived. Such *-algebras have often been encountered in mathematical physics, especially quantum field theory and quantum statistical mechanics [158, 160], and were first rigorously introduced by Borchers [52]. To develop the theory of these structures, several authors [11, 14, 21, 22, 25, 27, 30, 75, 76, 80, 82, 83, 87, 88, 89, 90] have recently embarked on the systematic extension of a number of notions and results in the theory of *-algebras, especially C^*- and W^*-algebras, to the more general partial *-algebraic context.

In the rest of this section, we outline some of the fundamental notions employed in the sequel.

2.1 Fundamental Notions:

2.1.1 Definition (Partial *-Algebra):

A *partial *-algebra* is a quadruplet $(\mathcal{A},^*, \cdot, \Gamma)$ consisting of a complex involutive linear space $\mathcal{A}$ with involution *, a partial multiplication $\cdot$, and a relation $\Gamma \subset \mathcal{A} \times \mathcal{A}$ on $\mathcal{A}$ such that

(i) $(x, y) \in \Gamma \Leftrightarrow (x \cdot y) \in \mathcal{A}$;

(ii) $(x, y), (x, z) \in \Gamma \Rightarrow (x, \alpha y + \beta z) \in \Gamma$; and in that case,

$x \cdot (\alpha y + \beta z) = \alpha(x \cdot y) + \beta(x \cdot z), \ \forall \ \alpha, \beta \in \mathbb{C}$, the complex numbers;

(iii) $(x, y) \in \Gamma \Leftrightarrow (y^*, x^*) \in \Gamma$; and in that case, $(x \cdot y)^* = y^* \cdot x^*$

2.1.2 Remark:

Let $(\mathcal{A},^*, \cdot, \Gamma)$ be a partial *-algebra. As only certain pairs of members of $\mathcal{A}$ may be multiplied, the operation $\cdot$ is , in general, nonassociative. Therefore, there is the need to isolate those members of $\mathcal{A}$ which can multiply a given element or set of elements. This leads us to the notion of multipliers.

2.2 Multipliers

Whenever $(x, y) \in \Gamma$, we say that x is a left multiplier of y and y a right multiplier of x, and write $x \in L(y)$ and $y \in R(x)$ respectively. By (ii), $L(x)$ and $R(x)$ are vector subspaces of $\mathcal{A}$. We also note that the partial multiplication $\cdot$, is distributive over addition in the following sense: if

$(x, y) \in \Gamma$ and $(x, z) \in \Gamma$ then

$$x \cdot y + x \cdot z = x \cdot (y + z)$$

Furthermore, for a subset $C \subseteq \mathcal{A}$, we shall use the following notation:

$$R(C) = \bigcap_{x \in C} R(x)$$

- *universal right multipliers* of every member of C;

$$L(C) = \bigcap_{x \in C} L(x)$$

- *universal left multipliers* of every member of C;

$$M(C) = L(C) \cap R(C)$$

- *universal multipliers* of every member of C.

In particular, the set

$$M(x) = L(x) \cap R(x)$$

consists of the *universal multipliers of* x.

Notice that the partial multiplication is not required to be associative, but it must be distributive with respect to the addition by (ii).

2.2.1 Remark: Employing the notion of a right or left multiplier; we can avoid any explicit reference to the relation Γ in the definition of a partial *-algebra. Obviously,

$$(x, y) \in \Gamma \Leftrightarrow x \in L(y) (\Leftrightarrow y \in R(x))$$

Therefore, we may replace the relation $(x, y) \in \Gamma$ by $x \in L(y)$ or $y \in R(x)$. We shall adopt this approach and refer to $(\mathcal{A},^*, \cdot)$ as a partial *-algebra.

2.2.2 Definition: A partial *-algebra $(\mathcal{A},^*, \cdot)$ is called unital if $\mathcal{A}$ contains an element e called a unit satisfying $e \in M(\mathcal{A}) = R(\mathcal{A}) \cap L(\mathcal{A})$, $e^* = e$ and $e \cdot x = x = x \cdot e$, $\forall x \in \mathcal{A}$. A unit of a unital partial *-algebra is unique [88].

2.2.3 Remark: As earlier mentioned that the partial multiplication of a partial *-algebra $\mathcal{A}$ is in general nonassociative. However, it is possible to endow it with a notion of associativity. One such definition is the following. A partial *-algebra $\mathcal{A}$ is associative if the following condition holds for any

$x, y, z \in \mathcal{A}$: whenever $x \in L(y)$, $y \in L(z)$ and $x \cdot y \in L(z)$, then $y \cdot z \in R(x)$ and one has

$$(x \cdot y) \cdot z = x \cdot (y \cdot z).$$

However, this condition is extremely restrictive and rarely realized in practice. There are, however, several interesting examples where a weaker form of associativity holds true. Therefore, we shall employ the following weaker concept.

2.2.4 Definition (Semiassociativity): A partial *-algebra $(\mathcal{A},^*, \cdot, \Gamma)$ is said to be *semiassociative* if $x, y \in \mathcal{A}$, with $y \in R(x)$ implies $y \cdot z \in R(x)$ for every $z \in R(\mathcal{A})$ and one has $(x \cdot y) \cdot z = x \cdot (y \cdot z)$.
This lack of associativity makes, for instance, the structure of abelian partial *-algebras much trickier than usual.

We remark that when $\mathcal{A}$ is semiassociative, then both $L(\mathcal{A})$ and $R(\mathcal{A})$ are algebras [but, in general, not *-algebras, since $L(\mathcal{A})^* = R(\mathcal{A})$ and $R(\mathcal{A})^* = L(\mathcal{A})$, where $\mathcal{C}^* = \{x^* : x \in \mathcal{C}\}$, for $\mathcal{C} \subseteq \mathcal{A}$].

To prove that

$$L(\mathcal{A})^* = R(\mathcal{A}) \;\text{ and }\; R(\mathcal{A})^* = L(\mathcal{A})$$

$$\text{Let } \; z \in R(\mathcal{A}), \text{ then } (x \cdot z)^* = z^* \cdot x^*, \;\; \forall\, x \in \mathcal{A}$$
$$= z^* \cdot y, \;\; \forall\, y \in \mathcal{A}.$$

Thus,

$$z \in R(\mathcal{A}) \;\;\Rightarrow\;\; z^* \in L(\mathcal{A})$$

$$\begin{aligned} &\Rightarrow \quad z \in L(\mathcal{A})^* \\ &\Rightarrow \quad R(\mathcal{A}) \subseteq L(\mathcal{A})^* \end{aligned}$$

Similarly, let $z^* \in L(\mathcal{A})^* \Rightarrow z \in L(\mathcal{A})$, then

$$\begin{aligned} (z \cdot x)^* &= x^* \cdot z^*, \ \ \forall\, x \in \mathcal{A} \\ &= y \cdot z^*, \ \ \forall\, y \in \mathcal{A} \end{aligned}$$

Thus,

$$\begin{aligned} z^* \in L(\mathcal{A})^* &\Rightarrow \quad z \in L(\mathcal{A}) \\ &\Rightarrow \quad z^* \in R(\mathcal{A}) \\ &\Rightarrow \quad L(\mathcal{A})^* \subseteq R(\mathcal{A}) \end{aligned}$$

Hence

$$R(\mathcal{A}) \subseteq L(\mathcal{A})^* \ \text{ and } \ L(\mathcal{A})^* \subseteq R(\mathcal{A})$$

altogether imply that $L(\mathcal{A})^* = R(\mathcal{A})$.

Also, one can show analogously that $R(\mathcal{A})^* = L(\mathcal{A})$. Therefore, $L(\mathcal{A})$ and $R(\mathcal{A})$ are not *-algebras, in general.

$M(\mathcal{A})$ is a *-algebra, since

$$\begin{aligned} M(\mathcal{A})^* &= (L(\mathcal{A}) \cap R(\mathcal{A}))^* = R(\mathcal{A})^* \cap L(\mathcal{A})^* = L(\mathcal{A}) \cap R(\mathcal{A}) \\ &= M(\mathcal{A}) \end{aligned}$$

For: $M(\mathcal{A})^* = (L(\mathcal{A}) \cap R(\mathcal{A}))^* \; = \; \{e^* : e \in L(\mathcal{A}) \text{ and } e \in R(\mathcal{A})\}$

$$= \{e^* : e \in L(\mathcal{A}) \cap R(\mathcal{A})\}$$

$$
\begin{aligned}
&= \{e^* : e \in R(\mathcal{A})^* \text{ and } e \in L(\mathcal{A})^*\} \\
&= \{e^* : e \in R(\mathcal{A})^* \cap L(\mathcal{A})^*\} \\
&= \{e^* : e \in (L(\mathcal{A}) \cap R(\mathcal{A}))^*\} \\
&= \{e : e^* \in L(\mathcal{A}) \cap R(\mathcal{A})\} \\
&= \{e : e^* \in M(\mathcal{A})\}
\end{aligned}
$$

This shows that $M(\mathcal{A})^* = (L(\mathcal{A}) \cap R(\mathcal{A}))^* = R(\mathcal{A})^* \cap L(\mathcal{A})^*$.

To prove that

$$M(\mathcal{A})^* = M(\mathcal{A})$$

Let $a \in M(\mathcal{A}) = L(\mathcal{A}) \cap R(\mathcal{A})$

$\Rightarrow a \in L(\mathcal{A})$ and $a \in R(\mathcal{A})$

$\Rightarrow a^* \in L(\mathcal{A})^*$ and $a^* \in R(\mathcal{A})^*$

$\Rightarrow a^* \in L(\mathcal{A})^* \cap R(\mathcal{A})^*$

$$
\begin{aligned}
\Rightarrow a \in (L(\mathcal{A})^* \cap R(\mathcal{A})^*)^* &= (R(\mathcal{A}) \cap L(\mathcal{A}))^* \\
&= M(\mathcal{A})^*
\end{aligned}
$$

$\Rightarrow M(\mathcal{A}) \subseteq M(\mathcal{A})^*$

Also, let $a \in M(\mathcal{A})^* = (R(\mathcal{A}) \cap L(\mathcal{A}))^*$

$\Rightarrow a^* \in R(\mathcal{A}) \cap L(\mathcal{A})$

$\Rightarrow a^* \in R(\mathcal{A})$ and $a^* \in L(\mathcal{A})$

$\Rightarrow a \in R(\mathcal{A})^*$ and $a \in L(\mathcal{A})^*$

$\Rightarrow a \in L(\mathcal{A})$ and $a \in R(\mathcal{A})$

$\Rightarrow a \in L(\mathcal{A}) \cap R(\mathcal{A}) = M(\mathcal{A})$

$\Rightarrow M(\mathcal{A})^* \subseteq M(\mathcal{A})$

Thus

$$a \in M(\mathcal{A}) \Rightarrow a \in M(\mathcal{A})^* \text{ and } a \in M(\mathcal{A})^* \Rightarrow a \in M(\mathcal{A}),$$

altogether imply that $M(\mathcal{A}) \subseteq M(\mathcal{A})^*$ and $M(\mathcal{A})^* \subseteq M(\mathcal{A})$

Whence, $M(\mathcal{A}) = M(\mathcal{A})^*$, showing that $M(\mathcal{A})$ is a *-algebra. This completes the proof.

2.2.5 Definition (Abelian Partial *-Algebra): A partial *-algebra $\mathcal{A}$ is said to be *abelian*, or *commutative*, if the following conditions hold.

(i) $(x, y) \in \Gamma \Leftrightarrow (y, x) \in \Gamma; \quad x, y \in \mathcal{A};$

(ii) $x \cdot y = y \cdot x, \quad (x, y) \in \Gamma.$

Equivalently,

(i') $R(x) = L(x), \forall\, x \in \mathcal{A}$, i.e. $y \in R(x)$ implies $y \in L(x)$.

(ii') $x \cdot y = y \cdot x, \forall x \in \mathcal{A}, y \in R(x)$.

2.3 Examples of Partial *-Algebras

We now give some examples of partial *-algebras.

2.3.1 Example: Partial *-algebras of functions

Let Ω be a Lebesgue-measurable set in $\mathbb{R}^n$, as usual, we denote by $L^p(\Omega)$, the Banach space of all measurable functions

$$f : \Omega \to \mathbb{C} \quad \text{such that} \quad \|f\|_p \equiv \left(\int_\Omega |f|^p dt \right)^{\frac{1}{p}} < \infty$$

For $f, g \in L^p(\Omega)$, consider the following set of real numbers:

$$\omega(f) = \{q \in [1, \infty) : \|f\|_q < \infty\}$$

We can now define a partial multiplication in $L^p(\Omega)$, taking Γ as

$$\Gamma = \{(f,g) \in L^p(\Omega) \times L^p(\Omega) : \exists r \in \omega(f), \quad \text{with}$$

$$pr(r-p)^{-1} \in \omega(g)\}$$

By Hölder's inequality, $(f,g) \in \Gamma$ implies that the (ordinary) product fg belongs to $L^p(\Omega) = \mathcal{A}$.

For: from $r \in \omega(f)$ and $pr(r-p)^{-1} \in \omega(g)$, we have

$$\frac{1}{r} + \frac{r-p}{pr} = \frac{p+r-p}{pr} = \frac{1}{p}$$

implying that $fg \in L^p(\Omega) = \mathcal{A}$.

Thus, $L^p(\Omega) = \mathcal{A}$ equipped with a partial multiplication is an algebra.

2.3.2 Remark: Ordinarily, $(f,g) \in L^p(\Omega) \times L^p(\Omega)$ implies that the pointwise product fg belongs to $L^{\frac{p}{2}}(\Omega)$. That is $L^p(\Omega)$ is not closed under ordinary multiplication, hence $L^p(\Omega)$ is not an algebra, since for each pair of elements $f, g \in L^p(\Omega)$, a product $fg \in L^p(\Omega)$ may not be defined.

But a partial multiplication can be defined, taking a relation

$$\Gamma \subset L^p(\Omega) \times L^p(\Omega) = \mathcal{A} \times \mathcal{A} \quad \text{as above}$$

2.3.3 Example: Partial *-algebras of polynomials

Let $\mathcal{B}(z)$ be the set of all complex polynomials of arbitrary degree in the real variable z. $\mathcal{B}(z)$ is an abelian *-algebra (where * is understood to be the complex conjugation), but it contains plenty of abelian partial *-algebras. Consider, for instance, a subset $\mathcal{B}_r(z)$ of $\mathcal{B}(z)$, defined by

$$\mathcal{B}_r(z) = \{p(z) \in \mathcal{B}(z) : \delta p \leq r\},$$

where we denote by δp the degree of the polynomial p. It is readily checked that $\mathcal{B}_r(z)$ is an abelian partial *-algebra with respect to

$$\Gamma = \{(p, q) \in \mathcal{B}_r(z) \times \mathcal{B}_r(z) : \delta p + \delta q \leq r\}$$

On the contrary, suppose Γ were not as defined above and we define an ordinary multiplication in $\mathcal{B}_r(z)$ as follows: If $p(z) \in \mathcal{B}_r(z)$, where $\delta p = r$ and $q(z) \in \mathcal{B}_r(z)$, where $\delta q = r$, $r > 0$; then $p(z).q(z) = v(z)$, say, where $\delta pq = \delta v = r + r = 2r > r$, $\forall r > 0$. That is the polynomial

$$v(z) = p(z) \cdot q(z) \in \mathcal{B}_{2r}(z)$$

$$\text{i.e., } \ v(z) = p(z) \cdot q(z) \notin \mathcal{B}_r(z).$$

Hence ordinary multiplication in $\mathcal{B}_r(z)$ is not closed.

2.3.4 Example: Topological quasi *-algebras [87].

Let $(\mathcal{A}_o, ^*, \cdot, \tau)$ be a topological *-algebra with involution *, product $\cdot$, and topology τ. By definition, the involution and all algebraic operations on $\mathcal{A}_0$ are τ-continuous. The τ-completion of the *-algebra $(\mathcal{A}_0, ^*, \cdot)$ is called a quasi *-algebra and will be denoted by $\mathcal{A}[\tau]$. In $\mathcal{A}[\tau]$, multiplication may no longer be continuous but one sees easily that $\mathcal{A}[\tau]$ is an $\mathcal{A}_0$-bimodule. The set Γ of pairs in $\mathcal{A}[\tau]$ for which products exist is given by

$$\Gamma = \{(x, y) \in \mathcal{A}[\tau] \times \mathcal{A}[\tau] : \text{ either } x \text{ or } y \text{ lies in } \mathcal{A}_0\}.$$

It follows that every quasi *-algebra has the structure of a partial *-algebra when we take $\mathcal{A}$ as the underlying linear space of $\mathcal{A}[\tau]$.

Let us give a concrete example: Take $\mathcal{A}_0 = C^0(\Omega)$, the space of continuous functions on a finite interval $\Omega \subset I\!R$, with pointwise multiplication and an L^p-norm $(1 \leq p < \infty)$. Then $\mathcal{A} = L^p(\Omega, dt)$ is an abelian, nonassociative, topological quasi *-algebra, and therefore $L(\mathcal{A}) = R(\mathcal{A}) = \mathcal{A}_0$.

2.3.5 Example: Infinite matrices and kernel

The space $\mathcal{M}_\infty$ of infinite matrices carries a very natural structure of partial *-algebra [96]: the multiplication of two matrices (a_{mn}), (b_{mn}) is well-defined iff

$$\sum_k a_{mk} b_{kn} < \infty.$$

In addition, $\mathcal{M}_\infty$ contains a relevant partial *-algebra, that of square matrices. This is the only one which may represent a partial *-algebra of closable operators.

In the same spirit, the space of distribution kernels [92], which can be identified with the space $\mathcal{D}'(\Omega \times \Omega)$ of distributions on $\Omega \times \Omega$, can be made into a partial *-algebra. The definitions, in this case, are trickier and involve certain families of spaces between $\mathcal{D}(\Omega)$ and $\mathcal{D}'(\Omega)$. In this case also one can select a partial *-algebra of distribution kernels which represent closable operators in $L^2(\Omega)$ [92].

Finally, a class of concrete partial *-algebras which are, in general, not quasi *-algebras may be described as follows: To each pair (D, H), consisting of a pre-Hilbert space D with completion H, whose inner product and norm are denoted by $< .,. >_H$ and $\| \cdot \|_H$, respectively, we associate the set $L^\dagger(D, H)$ of all linear maps A from D to H satisfying $\mathcal{D}(A^*) \supset D$, where $\mathcal{D}(T)$ denotes the domain of T and A^* is the adjoint of A. Then, $L^\dagger(D, H)$ is an involutive linear space when supplied with the usual notions of addition and scalar multiplication, and the involution

$$A \longmapsto A^\dagger = A^* \mid D, \quad A \in L^\dagger(D, H).$$

Furthermore, by defining Γ as

$$\Gamma = \{(A_1, A_2) \in L^\dagger(D, H) \times L^\dagger(D, H) : A_2 D \subset \mathcal{D}(A_1^{\dagger *}) \text{ and } A_1^\dagger D \subset \mathcal{D}(A_2^*)\}$$

a partial multiplication $\bullet$ is induced on $L^\dagger(D, H)$ if we define $A_1 \bullet A_2$ as $A_1^{\dagger *} A_2$ whenever $(A_1, A_2) \in \Gamma$. The quadruplet $(L^\dagger(D, H), ^\dagger, \bullet, \Gamma)$ is therefore a partial *-algebra, denoted in the sequel by $L^\dagger_\omega(D, H)$. This partial *-algebra, which is, in general, not a quasi *-algebra, has been studied by Antoine and his collaborators [21, 22, 25, 27] and also independently by Ekhaguere [75, 76, 77, 78, 79, 80, 81, 82, 83, 84, . . . , 91].

REFERENCES

[1] S. A. Ajupov(1977) : The ergodic theorem in O^*-algebras (Russian) Dokl. Akad. Naŭk uzs, no.3, 3-4.

[2] C.A. Akemann; J. Anderson; Gert K. Pedersen(1982) : Triangle inequalities in operator algebras. Linear and Multilinear Algebras, 11, no. 2, 167-178.

[3] S.A. Albeverio; R.J. Hϕegh-Krohn(1977): Dirichlet forms and Markov semigroups on $\mathcal{C}^*$-algebras. Comm. Math. Phys. 56, no. 2, 173-187.

[4] J. Alcantara; D.A. Dubin(1981): I^*-algebras and their applications. Publ. RIMS, Kyoto Univ. 17 , no. 1, 179-199.

[5] G.R. Allan(1967); Proc. London Math. Soc. (3). 17 , 91-114, especially pp. 91-94.

[6] W. Ambrose(1945); Trans. Amer. Math. Soc. 57 , 364-386.

[7] Frank W. Anderson; Kent R. Fuller (1992): Rings and Categories of modules (second edition) (Graduate Texts in Mathematics, 13) Springer-Verlag New York, Inc.

[8] J. -P. Antoine(1980): Partial inner product spaces. III. Compatibility relations revisited. J. Math. Phys. 21 , no. 2, 268-279.

[9] J. -P. Antoine(1980): Partial inner product spaces. IV. Topological considerations. J. Math. Phys. 21 , no. 8, 2067-2079.

[10] J. -P. Antoine(1978): Indefinite metric and Poincaré covariance. Mathematical aspects of quantum field theory, Part I (Karpacz, 1978), pp. 225-260, ACTA Univ. Wratislav, 519, Wydawn. Univ. Wroclaw; Wroclaw, 1979.

[11] J. -P. Antoine(1985): States and representations of partial *-algebras. In spontaneous symmetry breakdown and related subjects. World scientific, Singapore, pp. 247-267.

[12] J. -P. Antoine; A. Grossmann(1976): Partial inner product spaces. I. General properties. J. Functional Analysis 23 , No. 4, 369-378.

[13] J. -P. Antoine; A. Grossmann(1976): Partial inner product spaces. II. Operators. J. Functional Analysis 23 , No. 4, 379-391.

[14] J. -P. Antoine; A. Inoue(1990): Representability of invariant positive sesquilinear forms on partial *-algebras, Math. Proc. Camb. Phil. Soc. 108, 337–353.

[15] J. -P. Antoine; A. Inoue(1990): Strongly cyclic vectors for partial Op^*-algebras. Math. Nachr., 145 , 45-54.

[16] J. -P. Antoine; A. Inoue, C. Trapani (1999): O^*-dynamical systems and *-derivations of unbounded operator algebras. Math. Nachr., 204, 5-28.

[17] J. -P. Antoine; A. Inoue; C. Trapani(1996): Partial *-algebras of closable operators: A review. Reviews in Mathematical Physics. Vol. 8, No. 1, 1-42, World Sci. Publ. Co.

[18] J. -P. Antoine; A. Inoue; C. Trapani(2000): Biweights on Partial *-Algebras. Journal of Mathematical Analysis and Applications, 242, 164-190.

[19] J. -P. Antoine; A. Inoue; H. Ogi(1997): Standard generalized vectors for partial O^*-algebras. Ann. Inst. H. Poincaré Phys. Théor. 67, no. 3, 223-258.

[20] J. -P. Antoine; A. Inoue; H. Ogi (2000): Standard generalized vectors and *-automorphism groups of partial 0^*-algebras. Reports on Mathematical Physics, Vol. 45, No. 1, 39-66.

[21] J. -P. Antoine; F.D. Mathot(1987): Partial *-algebras of closed operators and their commutants I. General structure. Ann. Inst.H. Poincaré Phys. Théor. 46 , no. 3, 299-324.

[22] J. -P. Antoine; F.D. Mathot; C. Trapani(1987): Partial *-algebras of closed operators and their commutants II. Commutants and bicommutants. Ann. Inst. H. Poincaré, Phys. Théor. 46 , no. 3, 325-351.

[23] J. -P. Antoine; A. Inoue(1991): Normal forms on partial O^*-algebras. J. Math. Phys. 32 , no. 8, 2074-2081.

[24] J. -P. Antoine; A. Inoue; C. Trapani (1995): Spatiality of *-derivations of partial O^*-algebras. J. Math. Phys. 36 (7), 3743-3758.

[25] J. -P. Antoine; A. Inoue; C. Trapani(1990): Partial *-algebras of closable operators I. The basic theory and the abelian case. Publ. RIMS. Kyoto Univ. 26, 359-395 .

[26] J. -P. Antoine; A. Inoue; C. Trapani(1992): On regularity of the partial O^*-algebras generated by a closed symmetric operator. Publ. RIMS, Kyoto Univ. 28 , no. 5, 757-774.

[27] J. -P. Antoine; A. Inoue; C. Trapani(1991): Partial *-algebras of closable operators II. States and representations of partial *-algebras. Publ. RIMS. Kyoto Univ. 27, 399-430 .

[28] J. -P. Antoine; K. Gustafson(1981): Partial inner product spaces and semi-inner product spaces. Adv. in Math. 41 , no. 3, 281-300.

[29] J. -P. Antoine; M. Vause(1981): Partial inner product spaces of entire functions. Ann. Inst. H. Poincaré Sect. A. (N.S.), 35 , no. 3, 195-224.

[30] J. -P. Antoine; W. Karwowski(1985): Partial *-algebras of closed linear operators in Hilbert space. Publ. RIMS, Kyoto Univ. 21 , no. 1, 205-236. Add./Err. ibid. 22 (1986), 507-511.

[31] J. -P. Antoine; W. Karwowski(1983): Partial *-algebras of Hilbert space operators. Operator algebras, ideals and their applications in theoret-

ical physics. Proc. Int. Conf. Leipzig 1983, Teubner - Texte Math 67, 29-39 (1984).

[32] J. -P. Antoine; W. Karwowski(1989): Commuting normal operators in partial Op^*-algebras. Ann. Inst. H. Poincaré, 50 , 161.

[33] J. -P. Antoine; Y. Soulet; C. Trapani(1995): Weights on partial *-algebras. J. Math. Anal. Appl. 192, no. 3, 920-941.

[34] J. -P. Antoine; F. Bagarello, C. Trapani (1999): Topological partial*-algebras: Basic properties and examples. Reviews in Mathematical Physics, Vol. 11, No. 3, 267-302.

[35] J. -P. Antoine; F. Bagarello, C. Trapani (1998): Extension of representations in quasi*-algebras. Ann. Inst. Henr. Poincaré, Vol. 69, No. 2, 241-264.

[36] J. -P. Antoine(1977): Partial inner product spaces: another approach to unbounded operators. Proceedings of the International Conference on Operator Algebras, Ideals and their Applications in Theoretical Physics (Leipzig, 1977), pp. 279-289, Teubner, Leipzig, 1978.

[37] W. B. Arveson(1969): Acta Math. 123 , 141-224.

[38] R.G. Ascoli; G. Epifanio; A. Restivo(1974): Algebras of unbounded operators in scalar product spaces. Rivista Mat. Univ. Parma 3 , 21-32.

[39] R.G. Ascoli; G. Epifanio; A. Restivo(1970): On the mathemaical description of quantized fields. Comm. Math. Phys. 18 , 291-300.

[40] Robert B. Ash(1972): Measure, Integration and Functional Analysis. Academic Press, Inc. New York, 1972.

[41] F. Bagarello; C. Trapani(1994): States and representations of CQ^*-algebras. Ann. Inst. H. Poincaré, 61 , 103-133.

[42] F. Bagarello; A. Inoue; C. Trapani (2001): Unbounded C^*-Seminorms and *-Representations of Partial *-algebras. Zeitschrift fur Analysis und ihre Anwendungen. Journal for Analysis and its Applications, Vol. 20, No. 2, 295-314.

[43] D. Bakić and B. Guljaš (2001): Operators on Hilbert H^*-modules. Journal of Operator Theory, Vol. 46, 123-141.

[44] Bruce A. Barnes(1983): The properties *-regularity and uniqueness of C^*-norm in a general *-algebra. Trans. Amer. Math. Soc. 279 (1983), no. 2, 841-859.

[45] H. Behncke(1970): Comm. Pure Appl. Math. 23, 189-200.

[46] Sterling K. Berberian(1974): Lectures in Functional Analysis and Operator Theory (Graduate Texts in Mathematics Vol. 15) Springer-Verlag, New York Inc.

[47] Subhash J. Bhatt(1980): A note on generalized B^*-algebra. II. H.J. Indian Math. Soc. (N.S.), 44, no. 1-4, 285-290 .

[48] Subhash J. Bhatt(1982): On Jordan representations of unbounded operator algebras. Proc. Amer. Math. Soc. 84 , no. 3, 393-396.

[49] Subhash J. Bhatt(1979): A note on generalized B^*-algebra I. J. Indian Math. Soc. (NS), 43 , no. 1-4, 253-257.

[50] S.J. Bhatt, A. Inoue, H. Ogi (1998): On C^*-Spectral Algebras. Rendiconti Del Circolo Matematico Di Palermo Serie II, Suppl. 56, pp 207-213.

[51] S.J. Bhatt, A. Inoue, H. Ogi (1999): Admissibility of weights on non-normed *-algebras. Transactions of the American Mathematical Society, Vol. 351, No. 11, 4629-4656.

[52] H. J. Borchers(1975); In RCP 25 (Strasbourg), 22 (1975), 26, also in Quantum Dynamics: Models and Mathematics (L., Streit, ed.) Acta Phys. Austr. Supply, 16 (1976), 15.

[53] H. J. Borchers(1962): On structure of the algebra of field operators. Nuovo Cimento 24 , 214-236.

[54] H. J. Borchers, J. Yngvason(1975): On the algebra of field operators. The weak commutant and integral decomposition of states. Comm. Math. Phys. 42 , 231-252.

[55] H. N. Boyadžiev(1978): Partially ordered B^*-equivalent Banach algebras. Serdica 4 , no. 1, 12-18.

[56] O. Bratteli; D.W. Robinson(1979): Operator Algebras and Quantum Statistical Mechanics, Volumes I, II, Springer-Verlag, Berlin et al., (1979, 1981).

[57] Robert M. Brooks(1973): Some algebras of unbounded operators. Math. Nachr. 56 , 47-62.

[58] L. G. Brown (1977): Stable isomorphism of hereditary subalgebras of C^*-algebras. Pacific J. Math. 71, 335-348.

[59] L. G. Brown, P. Green and M.A. Rieffel (1977): Stable isomorphism and strong Morita equivalence of C^*-algebras. Pacific J. Math. 71, 349-363.

[60] J. Bunce and N. Salinas (1976): Completely positive maps on C^*-algebras and the left matricial spectra of an operator, Duke Math. J. Vol.43, No.4, 747-774.

[61] R.C. Busby (1968): Double centralizers and extensions of C^*-algebras, Trans. Amer. Math. Soc. 132, 79-99.

[62] C. E. Chidume(1995): Functional Analysis: Fundamental Theorems with applications. ICTP, Trieste, Italy.

[63] Man-Duen Choi and Edward G. Effros (1976): The completely positive lifting problem for C^*-algebras, Annals of Mathematics, vol.104, No.3, 585-609.

[64] Man-Duen Choi and Edward G. Effros (1977): Injectivity and operator spaces, Journal of Functional Analysis, Vol.24, 156-209.

[65] E. A. Coddington(1965). Canadian J. Math. 17 , 1030-1040.

[66] A. Connes(1973). Ann. Ecole. Norm. sup (4) 2 , 133-252.

[67] J.M Cook (1953): The Mathematics of Second Quantization. Trans. Amer. Math. Soc. 74, 222.

[68] G.B. Dell'Antonio; S. Doplicher and D. Ruelle (1966): Commun. Math. Phys. 2, 223-230.

[69] P. G. Dixon(1971): Unbounded operator algebras. Proc. London Math. Soc. 23, 53-69.

[70] N. Dunford; Jacob T. Schwartz(1958): Linear operators. Part I. General Theory, Interscience, New York .

[71] N. Dunford; Jacob T. Schwartz(1963): Linear operators. Part II Spectral Theory, Selfadjoint Operators in Hilbert Spaces. Interscience Publishers .

[72] N. Dunford; Jacob T. Schwartz (1971): Linear operators. Part III Spectral Operators, Wiley-Interscience .

[73] P. G. Dodds; B. de Pagter(1987): Algebras of unbounded scalar-type spectral operators. Pacific J. Maths 130 , no. 1, 41-74.

[74] G. O. S. Ekhaguere(1978): The theory of superselection rules I: A class of inequivalent, irreducible *-representations of the canonical commutation relations of the free electromagnetic field. J. Math. Phys. 19 , no. 8, 1751-1757.

[75] G. O. S. Ekhaguere(1985): Quantum stochastic integration in certain partial *-algebras. Stochastic processes in classical and quantum systems. (Ascona, 1985), 237-245, Lecture Notes in Phys. 262, Springer, Berlin, New York, 1986.

[76] G. O. S. Ekhaguere(1988): Dirichlet forms on partial *-algebras Math. Proc. Cambridge Philos. Soc. 104 , no. 1, 129-140.

[77] G. O. S. Ekhaguere(1979): Markov Fields in Noncommutative Probability Theory on W^*-algebras. J. Math. Phys. 20 (8) , 1679-1983.

[78] G. O. S. Ekhaguere(1982): On the Existence and Uniqueness of Noncommutative Stochastic Processes. J. Phys. A., Math. Gen., 15 , 3453-3463.

[79] G. O. S. Ekhaguere(1985): On Central Limit Theorems in Probability Gage Spaces. Publ. RIMS, Kyoto Univ. Vol. 21, No. 3.

[80] G. O. S. Ekhaguere(1989): Unbounded partial Hilbert algebras. J. Math. Phys. 30 (9) , 1957-1964.

[81] G. O. S. Ekhaguere(1992): Lipschitzian Quantum Stochastic Differential inclusions. Intern. Journal of Theoretical Physics. Vol. 31, No. 11, 2003-2027.

[82] G. O. S. Ekhaguere(1992): A noncommutative mean ergodic theorem for partial W^*-dynamical semigroups. ICTP, IC/92/410, (1992) Miramare - Trieste, 1-11.

[83] G. O. S. Ekhaguere(1991): Partial W^*-dynamical systems. Current topics in operator theory. World Scientific, New York, 202-217.

[84] G. O. S. Ekhaguere(1990): The Functional It$\hat{o}$ formula in quantum stochastic calculus. J. Math. Phys. 31 (12), 2921-2929.

[85] G. O. S. Ekhaguere(1983): On noncommutative stochastic integration I. J. Nig. Math. Soc. 1, 11-23.

[86] G. O. S. Ekhaguere(1987): On noncommutative stochastic integration II. J. Nig. Math. Soc. 6 , 13-32.

[87] G. O. S. Ekhaguere(1991): Unbounded quantum dynamical systems. Contemporary Stochastic Analysis, 13-29, edited by G.O.S. Ekhaguere, World Scientific, Singapore, New Jersey, London, Hong Kong.

[88] G. O. S. Ekhaguere(1996): Representation of completely positive maps between partial *-algebras, International Journal of Theoretical Physics, Vol. 35, No. 8, 1571-1580.

[89] G. O. S. Ekhaguere(1993): Noncommutative mean ergodic theorem on partial *-algebras, International Journal of Theoretical Physics 32; 1187-1196.

[90] G. O. S. Ekhaguere; P.O. Odiobala(1991): Completely positive conjugate-bilinear maps on partial *-algebras. J. Math. Phys. 32 (11)(1991), 2951-2958.

[91] G.O.S. Ekhaguere (2001): Algebraic Representation Theory of Partial Algebras, Annales Henri Poincaré, Vol. 2, No.2, 377–385.

[91]′ G.O.S. Ekhaguere(2002): Private communication.

[92] G. Epifanio; C. Trapani(1991): Partial *-algebras and Voltera convolution of distribution kernels. J. Math. Phys. 32 , 1096-1101.

[93] G. Epifanio; T. Todorov; C. Trapani (1992): Complete sets of compatible nonself-adjoint observables. Helv. Phys. Acta 65, 1-10.

[94] G. Epifanio; T. Todorov; C. Trapani(1996): Complete sets of compatible nonself-adjoint observables: an unbounded approach. J. Math. Phys. 37.

[95] G. Epifanio; C. Trapani(1984): V^*-algebras: a particular class of unbounded operator algebras. J. Math. Phys. 25 , no. 9, 2633-2637.

[96] G. Epifanio; C. Trapani(1988): Partial *-algebras of matrices and operators. J. Math. Phys. 29 , no. 3, 536-540.

[97] Maria Fragoulopoulou(1981): Spaces of representations and enveloping *l.m.c.* *-algebras. Pacific J. Math. 95 , no. 1, 61-73.

[98] K. Friedrichs (1953): Mathematical Aspects of the Quantum Theory of Fields, Interscience, New York.

[99] L. Gårding and A.S. Wightman (1964): Fields as Operator-valued Distributions in Quantum Field Theory. Ark. Fys. 28, No.13.

[100] I. M. Gel'fand; M. A. Naimark(1943): On the embedding of normed rings into the ring of operators in Hilbert space. Mat. Sb. 12 , 197-213.

[101] James G. Glimm and Richard V. Kadison (1960): Unitary operators in C^*-algebras, Pac. J. Math., Vol.10, No.2, 547-556.

[102] D.B. Goodner (1950): Projections in normed linear spaces, Trans. Am. Math. Soc. 69, 89-108, 1950.

[103] A. Grothendieck(1957): Un résultat sur le dual d'une (C^*-algebre, J. Math. Pures App. (9), 36 , 97-108.

[104] S. P. Gudder (1979): Pacific J. Math. 80, 14.

[105] S. P. Gudder; W. Scruggs(1977): Unbounded representations of *-algebras. Pacific J. Math. 70 , 369-382.

[106] R. Haag (1958): Quantum Field Theories with Composite Particles and Asymptotic Conditions, Phys. Rev. 112, 669.

[107] R. Haag; D. Kastler(1964): An Algebraic Approach to Quantum Field Theory. J. Math. Phys. 5, 848-861.

[108] Uffe Haagerup(1977): L^p-spaces associated with an arbitrary von Ne$\overset{c}{u}$mann algebra. Operator algebras and their applications in Mathematical Physics (Proc. Colloq., Marseille, (1977), pp. 175-184, CNRS, Paris, 1979.

[109] Paul R. Halmos (1950): Measure Theory, D. Van Nostrand Co., Inc., N.Y.

[110] Edwin Hewitt; Karl R. Stromberg (1965): Real and Abstract Analysis, Springer-Verlag, N.Y.

[111] E. Hille and R.S. Phillips (1957): Functional Analysis and Semigroups, American Mathematical Society Colloquium Publications Vol. 31.

[112] K.H. Hofmann (1972): Representation of algebras by continuous sections, Bull. Amer. Math. Soc. 78, 291-373.

[113] S. S. Horuzhy; A. V. Voronin(1988): Field algebras do not leave field domains invariant. Comm. Math. Phys. 102 , 687-692.

[114] R. L. Hudson; K.R. Parthasarathy(1984): Quantum It$\hat{o}$'s formula and stochastic evolutions. Comm. Math. Phys. 93 , no. 3, 301-323.

[115] T. Husain; B. D. Malviya(1972): Representations of locally m-convex *-algebras, Math. Japon 17 , 39-47.

[116] T. Husain; B. D. Malviya(1972): Colloq. Math. 24 (1971/72), 235-240.

[117] T. Husain; B.D. Malviya(1973): On semi-inner product spaces. II. Colloq. Math. 27 , 95-105.

[118] Atsushi Inoue(1978): L^p-spaces and maximal unbounded Hilbert algebras. J. Math. Soc. Japan 30 , no. 4, 667-686.

[119] Atsushi Inoue(1976): On a class of unbounded operator algebras. II. Pacific J. Math. 66, no. 2, 411-431.

[120] Atsushi Inoue(1978): A class of unbounded operator algebras. IV. J. Math. Anal. Appl. 64 , no. 2, 334-347.

[121] Atsushi Inoue(1976): On a class of unbounded operator algebras. I. Pacific J. Math. 65 (1976), no. 1, 77-95.

[122] Atsushi Inoue(1977): On a class of unbounded operator algebras. III. Pacific J. Math. 69, no. 1, 105-115.

[123] Atsushi Inoue(1995): Standard generalized vectors for algebras of unbounded operators. J. Math. Soc. Japan, Vol. 47, No. 2.

[124] Atsushi Inoue, W. Karwowski (1999): Standard Weights on algebras of unbounded operators. J. Math. Soc. Japan, Vol. 51, No. 4, 911-935.

[125] Atsushi Inoue(1983): On regularity of positive linear functionals. Japan J. Math. (N.S.) 9 , no. 2, 247-275.

[126] Atsushi Inoue, Klaus-Detlef Kürsten (2000): Trace Representation of Weights on Algebras of Unbounded Operators. Journal of Mathematical Analysis and Applications, 247, 136-155.

[127] Atsushi Inoue(1988): A generalization of Hilbert algebras by partial *-algebras. Bull. Central Res. Inst. Fukuoka Univ. No. 104 , 57-69.

[128] Atsushi Inoue(1987): An unbounded generalization of the Tomita-Takesaki theory. II. Publ. RIMS, Kyoto Univ. 23, no. 4, 673-726.

[129] Atsushi Inoue(1986): An unbounded generalization of the Tomita-Takesaki theory. I, Publ. RIMS, Kyoto Univ. 22, no. 4, 725-765.

[130] Atsushi Inoue(1991): Partial O^*-algebras generated by two closed symmetric operators. J. Oper. Theory 26 , 93-108.

[131] Atsushi Inoue(1983): J. Operator Theory 10, 77 .

[132] Atsushi Inoue(1991): Extension of unbounded left Hilbert algebras to partial *-algebras. J. Math. Phys. 32 , 323-331.

[133] Atsushi Inoue; S. $\hat{O}$ta(1980): Derivations on algebras of unbounded operators. Trans. Amer. Math. Soc. 261 , no. 2, 567-577.

[134] Atsushi Inoue; Hideyuki Ueda; T. Yamauchi(1987): Commutants and bicommutants of algebras of unbounded operators. J. Math. Phys. 28, no. 1, 1-7.

[135] Atsushi Inoue; Kunimichi Takesue(1983): Self-adjoint representations of polynomial algebras. Trans. Amer. Math. Soc. 280 , no. 1, 393-400

[136] Atsushi Inoue; S. $\hat{O}$ta; J. Tomiyama(1981): Derivations of operator algebras into spaces of unbounded operators. Pacific J. Math. 96, no.2, 389-404.

[137] Atsushi Inoue; Hidekazu Ogi (1998): Regular weights on algebras of unbounded operators. J. Math. Soc. Japan, Vol. 50, No.1, 227-252.

[138] B.E. Johnson (1964): An Introduction to the theory of centralizers. Proc. London Math. Soc., 14, 299-320.

[139] B.E. Johnson (1964): Centralizers on certain topological algebras. Journal London Math. Soc., 39, 603-614.

[140] B.E. Johnson (1968): Centralizers and operators reduced by maximal ideals, J. London Math. Soc. 43, 231-233.

[141] B.E. Johnson (1966): Continuity of centralizers on Banach algebras. Journal London Math. Soc., 41, 639-640.

[142] David L. Johnson.; C. D. Lahr(1979): Dual A^*-algebras of the first kind. Proc. Amer. Math. Soc. 74 , no. 2, 311-314.

[143] David L. Johnson.; C. D. Lahr(1979): Multipliers of A^*-algebras. Proc. Amer. Math. Soc. 74 , no. 2, 315-317.

[144] R. V. Kadison; J. R. Ringrose(1983): Fundamentals of the theory of operator algebras I. Academic Press, New York, London .

[145] Irving Kaplansky(1951): Projections in Banach algebras, Annals of Mathematics, vol. 53 , pp 235-249.

[146] Irving Kaplansky (1953): Modules over Operator algebras, Amer. J. Math. Vol. 75, 839-858.

[147] Witold Karwowski(1991): Abelian partial *-algebras: the case of normal commuting operators. Current topics in operator algebras (Nara 1990), 297-303. World Sci. Publ. River Edge, N.J.

[148] G.G. Kasparov (1980): Hilbert C^*-modules: Theorems of Stinespring and Voiculescu, J. Operator Theory Vol. 4, 133–150.

[149] T. Kato(1966): Perturbation Theory for Linear Operators. Springer-Verlag, Berlin, New York.

[150] J. L. Kelley; R.L. Vaught(1953): The positive cone in Banach algebras. Trans. Amer. Math. Soc. 74 , 44-55.

[151] H. Kosaki(1985); Amer. J. Math. 107 , no. 3, 697-735.

[152] S. G. Krein(1972): Linear Differential Equations in Banach space (Translations of mathematical monographs. Vol. 29), American Mathematical Society, USA .

[153] Erwin Kreyszig(1978): Introductory Functional Analysis with Application. John Wiley and Sons, Inc. New York.

[154] K. D. Kürsten, E. Wagner (2000): On representations of partial *-algebras based on $\mathcal{B}$-weights. Z. Anal. Anwendungen 19, no. 3, 623-638.

[155] Jesus Gil de Lamadrid(1984): Extending positive definite linear forms. Proc. Amer. Math. Soc. 91 , no. 4, 593-594.

[156] C. Lance (1976): Tensor products of non-unital C^*-algebras, J. London Math. Soc., Vol. 12, part 2, 160-168.

[157] G. Lassner(1972): Topological algebras of operators. Rep. Mathematical Phys. 3 , No. 4, 279-293.

[158] G. Lassner(1984): Algebras of unbounded operators and quantum dynamics. Physica A 124 , 471 - 480.

[159] G. Lassner; A. Uhlmann(1975): On $Op*$-algebras of unbounded operators. International Conference on the Mathematical Problems of Quantum Field Theory and Quantum Statistics. Part I. Axiomatic Quantum Field Theory. Trudy Math. Inst. Steklov, 135 (1975), 168, 173, 259-260.

[160] G. Lassner; G. A. Lassner(1977); Completely positive mappings and unbounded observables. Rep. Math. Phys. 11, 133-140.

[161] Frank, Löffler, W. Timmermann(1986): Singular states on maximal Op^*-algebras. Publ. RIMS, Kyoto Univ. 22, No. 4, 671-687.

[162] R. J. Loy(1976); J. London Math. Soc. (2), 14 , No. 3, 423-429.

[163] G. Lumer(1961); Trans. Amer. Math. Soc. 100 , 29-43.

[164] L. M$\acute{a}t\acute{e}$(1967); Studia Math. 28 (1960/67), 227-234.

[165] Francoise D. Mathot(1981): Distributionlike representation of *-algebras. J. Math. Phys. 22 , no. 7, 1386 - 1389.

[166] Paul-Andre Meyer(1993): Quantum Probability for Probabilists. Springer-Verlag, Berlin Heidelberg .

[167] Noor Mohammad; A. B. Thaheem(1991): P-Commutative topological *-algebras. ICTP IC/91/199, Miramare -Trieste.

[168] Noor Mohammad; A. B. Thaheem(1988): A Hilbert space structure on Banach algebras ICTP IC/88/222, Miramera - Trieste.

[169] Noor Mohammad; Alberto Verjovsky(1991): On an inner product over lmc *-algebras. ICTP IC/91/218, Miramare - Trieste .

[170] Noor Mohammad; S. Somasundaram(1991): Characterizations of locally C^*-algebras. ICTP IC/91/245, Miramare Trieste.

[171] R.L. Moore (1932): Foundations of point set theory. American Math. Soc. Colloquium Publications, Vol. 13, New York.

[172] R. T. Moore(1969): Bull. Amer. Math. Soc. 75 , 68-73.

[173] G.D. Mostow (1961): Cohomology of topological groups and solvmanifolds, Annals of Math. Vol.73, No.1, 20-48.

[174] Tetsuya Musuda(1983): L_p-spaces for von Ne$\overset{c}{u}$mann algebra with reference to a faithful normal semi-finite weight. Publ. Res. Inst. Sci. 19 , no. 2, 673-727.

[175] F.J. Murray, J. von Neumann(1936): On rings of operators. Annals of Maths. vol. 37, No.1, 116-229, January 1936.

[176] F.J. Murray(1936): Linear transformations in $\mathcal{L}_p$, $p < 1$ Trans. Amer. Math. Soc. vol. 39, pp 83-100.

[177] F. J. Murray (1937): On complementary Manifolds and projections in spaces L_p and l_p, Trans. Amer. Math. Soc. Vol. 41 , pp 138-152.

[178] F. J. Murray (1942): The analysis of linear transformations, Bull. Amer. Math. Soc. Vol. 48 , pp 76-93.

[179] F. J. Murray (1945): Quasi-complements and closed projections in reflexive Banach Spaces. Trans. Amer. Math. Soc. 58, 77-95 .

[180] I. S. Murphy(1969): Bull. London Math. Soc. 1 , 171-173.

[181] M. A. Naimark(1972): Normed Algebras. Wolters-Noordhoff Publishing Groningen, The Netherlands.

[182] E. Nelson(1959): Analytic vectors. Ann. Math. (2) 70 , 572-615.

[183] J. von Neumann(1936): On a certain topology for rings of operators. Annals of Maths. Vol. 37, No. 1, 111-115, January, 1936.

[184] Tamio Ono(1969): Note on a B^*-algebra II. Bull. Nagoya Inst. Tech. 21 , 93-95.

[185] Tamio Ono(1959): J. Math. Soc. Japan II , 146-158.

[186] Tamio Ono(1967): Polar decomposition in a B^*-algebra. Bull. Nagoya Inst. Tech. 19 , 111-113.

[187] T. Onwusu-Ansah(1977): Extending Hilbert space results to Banach spaces. Proceedings of the international symposium on Functional Analysis and its Applications. (Edited by: A. Olubummo; S.O. Iyahen; G.O.S. Ekhaguere) Ibadan, 1977, p. 127-149.

[188] O. Ore(1944): Trans. Amer. Math. Soc. 55 , 493-513.

[189] W. Page(1973): Characterizations of B^* and A^*-algebras. Rev. Roumaine Math. Pures Appl. 18, 1241-1244.

[190] T. W. Palmer(1972): Hermitian Banach *-algebras. Bull. Amer. Math. Soc. 78 , 522 - 524.

[191] T.W. Palmer (1970): Real C^*-algebras, Pac. J.Math. Vol. 35, No. 1, 195-204.

[192] K.R. Parthasarathy(1992): An Introduction to Quantum Stochastic Calculus l. Monographs in Mathematics; vol.85, Basel, Boston, Berlin.

[193] W. L. Paschke(1973): Inner product modules over B^*-algebras; Trans. Amer. Math. Soc. 182, 443-468 .

[194] Gert K. Pedersen(1984): Multipliers of AW^*-algebras. Math. Z. 187 , no. 1, 23-24.

[195] Gert K. Pedersen(1989): Analysis Now (Graduate Texts in Mathematics, 118) Springer-Verlag, New York Inc.

[196] Gert K. Pedersen(1979): C^*-Algebras And Their Automorphism Groups. Academic Press, London, New York, San Francisco.

[197] R. T. Powers(1974): Self-adjoint algebras of unbounded operators. II. Trans. Amer. Math. Soc. 187 , 261-294.

[198] R. T. Powers(1971): Self-adjoint algebras of unbounded operators I., Comm. Math. Phys. 21 , 85-124.

[199] Eduard Prugovečki(1981): Quantum Mechanics in Hilbert space, second edition (Pure and Applied Mathematics, a series of monographs and text books: V. 92). Academic Press, Inc. USA.

[200] M. Reed; B. Simon(1972): Methods of modern Mathematical Physics. Volume I: Functional Analysis. Academic Press, New York and London.

[201] C. E. Rickart(1960): General theory of Banach algebras, Van Nostrand, Princeton, N. Y.

[202] C. E. Rickart(1946): Ann. of Math. 2 (47) (1946), 528 - 550.

[203] M.A. Rieffel (1974): Induced representations of C^*-algebras, Advances in Math. Vol. 13, 176-257.

[204] M.A. Rieffel (1974): Morita equivalence for C^*-algebras and W^*-algebras. J. Pure Appl. Algebra 5, 51-96.

[205] Walter Rudin(1974): Real and Complex Analysis Tata McGraw-Hill, Inc. New Delhi.

[206] Russo and H.A. Dye (1966): A note on unitary operators in C^*-algebras, Duke Math. J. Vol. 33, No. 2, 413-416.

[207] S. Sakai(1971): C^*-Algebras and W^*-Algebras, Springer-Verlag.

[208] H. H. Schaefer(1966): Topological vector spaces. The Macmillan Company, New York.

[209] Konrad Schmüdgen(1983): Graded and filtrated topological *-algebras I. Graded normal topologies. Rep. Math. Phys. 18 , no. 2, 211-229.

[210] Konrad Schmüdgen(1988): Spaces of continuous sesquilinear forms associated with unbounded operator algebras. Z. Anal. Anw., 7 , 309-319.

[211] Konrad Schmüdgen(1984): Graded and filtrated Topological *-algebras. II. The closure of the positive cone. Rev. Roumaine Math. Pures Appl. 29 , No. 1, 89-96.

[212] Konrad Schmüdgen(1986): On commuting unbounded self-adjoint operators IV. Math. Nachr. 125 , 83-102.

[213] Konrad Schmüdgen(1985): On commuting unbounded selfadjoint operators III. Manuscripta Math. 54 , no. 1-2, 221-247.

[214] Konrad Schmüdgen(1984): On commuting unbounded selfadjoint operators I. Acta. Sci. Math. (Szeged) 47 , No. 1, 131-146.

[215] Konrad Schmüdgen(1983): On the Heisenberg commutation relation. II. Publ. RIMS, Kyoto Univ. 19 , no. 2, 601-671.

[216] Konrad Schmüdgen(1987): Unbounded commutants and intertwining spaces of unbounded symmetry operators and *-representations. J. Funct. Anal. 71 , no. 1, 47-68.

[217] Konrad Schmüdgen(1985): A formally normal operator having no normal extension. Proc. Amer. Math. Soc. 95 , No. 3, 503-504.

[218] Konrad Schmüdgen(1986): A note on commuting unbounded self-adjoint operators affiliated to properly infinite von Neumann algebras. II. Bull London Math. Soc. 18 , no. 3, 287-292.

[219] Konrad Schmüdgen(1988): Strongly commuting self-adjoint operators and commutants of unbounded operator algebras. Proc. Amer. Math. Soc. 102 , No. 2, 365 - 372.

[220] Konrad Schmüdgen , J. Friedrich(1984): On commuting unbounded self-adjoint operators. II. J. Integral Equation Operator Theory, 7, No. 6, 815-867.

[221] K. Schmüdgen(1978): Math. Nachr. 85 , 161-170 .

[222] K. Schmüdgen(1975): Der beschränkte Beitraage zur Theorie nicht-normierbarer topologischer Algebren. Wiss. Z. Karl-Marx-Univ. Leipzig Math-Natur. Reihe 24 , No. 5, 473-490.

[223] K. Schmüdgen(1985): A note on Commuting unbounded selfadjoint operators affiliated to properly infinite von Neu̒mann algebras I. Bull. London Math. Soc. 17 , no. 1, 52-54.

[224] K. Schmüdgen(1983): On domains of powers of closed symmetric operators. J. Oper. Theory. 9 , 53-75.

[225] K. Schmüdgen(1990): Unbounded Operator Algebras and Representation Theory. Akademie - Verlag.

[226] Z. Sebestyen(1973): A weakening of the definition of C^*-algebras. Acta Sci. Math. (Szeged) 35 , 17-20.

[227] I. E. Segal(1947): Irreducible representations of operator algebras. Bull. Amer. Math. Soc. 61 , 69 - 105.

[228] I. E. Segal(1963): Mathematical problems of relativistic physics. Amer. Math. Soc., Providence, R.I.

[229] I.E. Segal (1956): Tensor Algebras over Hilbert Spaces. Trans. Amer. Math. Soc. 81, 106-134.

[230] J. Shabani(1987): On some classes of unbounded commutants of unbounded operator families. J. Math. Phys. 28 , No. 9, 2046-2050.

[231] J. Shabani(1988): On some classes of Topological quasi *-algebras. J. Ann. Inst. H. Poincaré. 48 , 97.

[232] S. Shirali; J.W.M. Ford(1970): Duke Math. J. 37 , 275 - 280.

[233] George F. Simmons: Topology and Modern Analysis. McGraw-Hill Book Company, Inc. (Int. Student Edition), New York. Tokyo.

[234] B. Simon(1971): Quantum mechanics for Hamiltonians defined as quadratic forms, Princeton Univ. Press, Princeton, N.Y.

[235] James F. Smith(1972): The p-classes of an H^*-algebra. Pacific J. Math. 42 , 777-793.

[236] W. F. Stinespring(1955): Positive functions on C^*-algebras. Proc. Amer. Math. Soc. 6, 211-216 .

[237a] E. Stϕrmer(1965): Trans. Amer. Math. Soc. 120 , 438-447, theorem 3.3.

[237b] E. Stϕrmer(1974): Positive Linear Maps of C^*-algebras, in Lecture Notes in Physics Vol. 29, Springer-Verlag, Berlin-Heidelberg, N.Y.

[238] R.F. Streater(1972): Mathematics of contemporary physics. Academic Press, London .

[239] R. F. Streater; A.S. Wightman(1964): PCT, Spin and statistics and All That. Benjamin, New York.

[240] M. Takesaki(1970): Tomita's theory of modular Hilbert algebras and its applications. Lecture Notes in Mathematics, Vol. 128, Springer, Berlin.

[241] Wayne Tiller(1973): P-commutative Banach *-algebras. Trans. Amer. Math. Soc. 180 , 327-336.

[242] T. S. Todorov(1976): Discussion of a theory of R. T. Powers. Bulgar. J. Phys. 3 , No 3, 266 - 269.

[243] Bohdan J., Tomiuk(1979): On some properties of Segal algebras and their multipliers. Manuscripta Math. 27 , No. 1, 1-18.

[244] C. Trapani(1988): States and derivations on quasi-*-algebras. J. Math. Phys. 29 , 1885 - 1890.

[245] N. T. Varopoulos(1964): C. R. Acad Sci. 258 , 2465-2467.

[246] M. H. Vasavada(1980): Generalizations of B^*-algebras. Mat. Bilten 29 (30) (1979/80), no. 3-4.

[247] D. Voiculescu (1976): A non-commutative Weyl-von Neumann theorem, Revue Roumaine De Mathematiques Pures et Appliquees, Tome XXI, No.1, 97-113.

[248] J.G. Wendel (1952): Left centralizers and isomorphisms of group algebras, Pacific J. Math., 2, 251-261.

[249] J.D. Weston (1960): Positive perfect operators, Proc. London Math. Soc. (3)10, 545-565.

[250] Pakken Wong(1973): *-actions in A^*-algebras. Pacific J. Math. 44 , 775 - 779.

[251] A. W. Wood(1977): Proc. London Math. Soc. (3), 34 , No. 2, 245 - 268.

[252] James Wood(1979): Unbounded multipliers on commutative Banach algebras. Pacific J. Math. 85 , No. 2, 479-484.