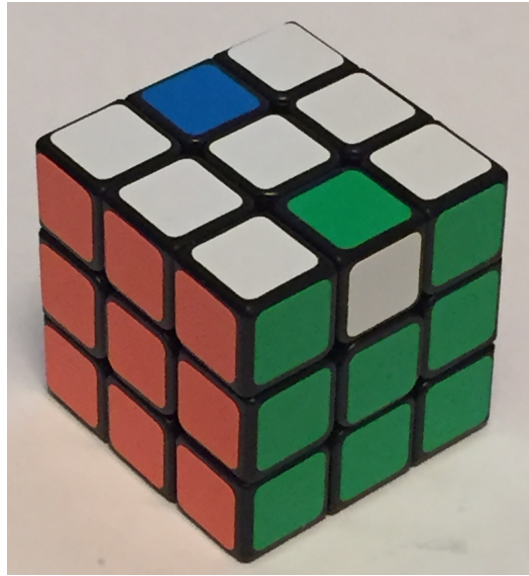# Rubik's cube edge processes

Tony Forbes

Introduction

It is assumed you are familiar with the basic properties of Rubik's cube—what it looks like, the way it works, and the main problem to be solved. You do not have to know how to solve the cube, and what we are not going to do here is offer a solution.

Our main purpose is to examine in some detail certain types of Rubik cube edge processes. First, we introduce some notation, recalling much of which from David Singmaster's *Notes*, [1].

Upper-case letters

$$U, D, F, B, L \text{ and } R$$

denote either the faces or the outer slices of the cube: up, down, front, back, left and right, respectively. This is David Singmaster's terminology [1]; his choice of face descriptions is nice because it provides six unique initial letters.

Lower-case letters subscripted with stars

$$u_*, d_*, f_*, b_*, l_* \text{ and } r_*$$

denote the inner slices adjacent to outer slices U, D, F, B, L and R, respectively. There is some redundancy here since both u_* and d_* represent the same slice; similarly for $\{f_*, b_*\}$ and for $\{l_*, r_*\}$. The actual slice identified by a letter depends on how you look at the cube. If you turn it upside down, then U becomes D.

A single piece of the cube is denoted by the letters of the outer slices that intersect it. For example: UFL, or any other permutation of these letters, is the piece in the up-front-left corner; UF or FU refers to the edge piece on the up-front edge; and U refers to the face centre piece of the up face. The same notation is also used for positions.

Face centre pieces {U, D, F, B, L, R} can be regarded as being in fixed positions. However, they do rotate. If inner slice turns (see below) are used, they can be permuted amongst themselves.

Edge pieces can only be permuted amongst themselves. However, an edge piece has two orientations. For example, if edge piece FU is in position FU, its correct place in the cube, it is either correctly orientated, in the sense that the F part of FU has the same colour as piece F, or it is flipped, i.e. the F part of FU has the colour of U.

Corner pieces can only be permuted amongst themselves. A corner piece in its correct position can have three distinct orientations; it might be correctly orientated, or twisted 120° clockwise, or twisted 120° anticlockwise.

The term *outer slice* refers to an entire assembly of 9 pieces: a face centre piece together with the 4 edge pieces and the 4 corner pieces adjacent to it. For example, outer slice F consists of {F, FU, FD, FL, FR, FUL, FUR, FDL, FDR}. The word *face* refers either to an actual face of the cube, or to an outer slice, the correct interpretation being obvious from the context.

The term *inner slice* refers to an entire assembly of 8 pieces sandwiched between two parallel outer slices. For example, inner slice f_* (or b_*) consists of {U, D, L, R, UL, UR, DL, DR}. An inner slice turn appears to move the face centre pieces. However, you can consider the rotated slice fixed and the rest of the cube rotated backwards.

Each of the symbols $\{U, D, F, B, L, R, u_*, d_*, f_*, b_*, l_*, r_*\}$ is also used to denote a 90° clockwise turn of the cube slice of that name. The dual meaning of these symbols would be confusing if the correct interpretation were not obvious from the context. Multiple turns are indicated by exponents modulo

4, and we usually write X' for X^{-1}. For example, F^2 means turn the F outer slice through 180°, and u'_* means turn the u_* slice 90° anticlockwise. For the inner slice turns, we have $u'_* = d_*$, $f'_* = b_*$ and $l'_* = r_*$.

The inverse, $\mathcal{P}' = \mathcal{P}^{-1}$, of a sequence of moves $\mathcal{P}$ is the sequence of inverses of the moves of $\mathcal{P}$ but in the reverse order. Given two sequences of moves $\mathcal{P}$ and $\mathcal{Q}$, the *conjugate* $(\mathcal{P}, \mathcal{Q})$ and *commutator* $[\mathcal{P}, \mathcal{Q}]$ are defined by

$$(\mathcal{P}, \mathcal{Q}) = \mathcal{P}\mathcal{Q}\mathcal{P}', \qquad [\mathcal{P}, \mathcal{Q}] = \mathcal{P}\mathcal{Q}\mathcal{P}'\mathcal{Q}'.$$

Observe that $(\mathcal{P}, \mathcal{Q})' = (\mathcal{P}, \mathcal{Q}')$ and $[\mathcal{P}, \mathcal{Q}]' = [\mathcal{Q}, \mathcal{P}]$.

The permutation corresponding to a sequence of moves is computed on the assumption that the face centre pieces are in their home positions. For example, the permutation corresponding to f_* has 2 corner 4-cycles and 2 edge 4-cycles.

There is a slightly overwhelming case for banning inner slices turns. They are not needed, since anything you can do to the cube can be achieved with $\{U, D, F, B, L, R\}$. However, many sequences described in [1] and elsewhere can be rewritten with significant reduction in clumsiness if inner slice turns are allowed. For example, Thistlethwaite's 4-flip $(R^2B^2R^2U^2RL', B)U$, [1, page 44], has this compact alternative representation: $[F^2l_*F^2l_*^2, U']$.

It is interesting to measure sequences by the number of moves they contain. We count any non-trivial power of a slice turn as a single move. So, for instance, the 6-flip sequence

$$[BF^2U^2L'FD^2RDF, f_*d_*l_*^2]$$

has $12 \times 2 = 24$ moves. Some writers count inner slices with weight 2 because they do a move such as f_* in two stages, $(Ff_*)\, F'$. However, I think it is natural and sensible to treat inner and outer slices as equals. Using the left hand, grip the F slice of a Rubik cube with the thumb and 4th finger and the B slice with the 1st and 3rd fingers. It is now easy to rotate the f_* slice with the right hand.

Edge processes $[X_1X_2 \ldots X_p, y_1y_2 \ldots y_q]$

We are interested in Rubik cube processes of the form

$$[\mathcal{X}, \mathcal{Y}],$$
$$\mathcal{X} = X_1X_2 \ldots X_p, \quad X_i \in \{F, B, L, R, U, D\}, \quad i = 1, 2, \ldots, p, \qquad (1)$$
$$\mathcal{Y} = y_1y_2 \ldots y_q, \quad y_j \in \{f_*, b_*, l_*, r_*, u_*, d_*\}, \quad j = 1, 2, \ldots, q.$$

3

Thus $[\mathcal{X}, \mathcal{Y}]$ is a sequence of face turns commutated with a sequence of inner slice turns. Recall that $[\mathcal{X}, \mathcal{Y}] = \mathcal{X}\mathcal{Y}\mathcal{X}'\mathcal{Y}'$. Since $\mathcal{Y}$ does not move corners, the corners moved by $\mathcal{X}$ are undone by $\mathcal{X}'$. Similarly, face centres moved by $\mathcal{Y}$ are restored by $\mathcal{Y}'$ since $\mathcal{X}$ has no action on them. Therefore $[\mathcal{X}, \mathcal{Y}]$ moves only edges; it is an edge process.

We want to know what cycle structures can be created by these processes. For definiteness, when we consider the length $|C|$ of an edge cycle C, we are assuming C is acting on the 24 *edge facelets*. However, the length of C when acting on the 12 *edge pieces* will be $|C|$ or $|C|/2$.

For example, $[RDUB, l_*u_*^2]$ acting on edge facelets has cycle structure

(FU, UB, BD, DF) (LU, DR)

(FR, LD, RF, DL) (BL, RU, LB, UR) (FL, LF) (BR, RB),

three 2-cycles and three 4-cycles. Moreover, we see that the last four cycles are *twisted* (Singmaster's terminology, [1]). If the cycle length is $2c$, the symbol at position $c + 1$ is just the first symbol repeated but reversed. In particular, the last two 2-cycles each act on a single edge piece by flipping it. Using the notation of [1], we can represent a twisted edge cycle by truncating just before the repeated symbol and adding a plus superscript,

(FU, UB, BD, DF) (LU, DR) (FR, LD)$^+$ (BL, RU)$^+$ (FL)$^+$ (BR)$^+$.

When acting on edge pieces the cycle lengths are therefore 4, 2, 2, 2, 1, 1; ten pieces move and two pieces are just flipped.

We wish to express these cycle structures compactly. Suppose $\mathcal{A}$ is an edge process, i.e. a sequence of moves that leaves the corners and face centres fixed. We adopt the notation

$$(s; a_1, a_2, \ldots, a_r),$$

where s is the number of edge pieces moved or flipped by $\mathcal{A}$, and for $i = 1, 2, \ldots, r$, a_i, is a number representing the length of a cycle of $\mathcal{A}$ as acting on cube facelets, decorated with a hat if that cycle is twisted.

For instance, the cycle pattern of the permutation generated by the above example, $[RDUB, l_*u_*^2]$, is $(12; \tilde{2}, \tilde{2}, 2, \tilde{4}, \tilde{4}, 4)$. We can compute s by summing the a_i with weight $1/2$ for twisted cycles and 1 for normal cycles. Here, $s = 2/2+2/2+2+4/2+4/2+4 = 12$, and we save the reader the trouble of doing the computation by including this number in the cycle pattern specification.

4

Let

$$A = (a_1, a_2, \ldots, a_r), \quad a_1, a_2, \ldots, a_r \geq 2,$$

$$E = (e_1, e_2, \ldots, e_r), \quad e_i \in \left\{ \frac{1}{2}, 1 \right\}, \quad i = 1, 2, \ldots, r, \tag{2}$$

$$s = E \cdot A = e_1 a_1 + e_2 a_2 + \cdots + e_r a_r.$$

We assume the elements of A are the lengths of the non-trivial cycles of an edge process $\mathcal{A}$ of the form $[\mathcal{X}, \mathcal{Y}]$ as in (1). We consider an element a_i of A as the length of a cycle, which is twisted iff $e_i = 1/2$, $i = 1, 2, \ldots, r$. Note that s is the number of edge pieces acted on nontrivially by $\mathcal{A}$. The following conditions must be satisfied.

(i) Since the cube has 12 edge pieces, each of which has 2 edge facelets, we must have $a_1, a_2, \ldots, a_r \leq 24$ and $s \leq 12$.

(ii) If a_i is odd, then $e_i = 1$. Twisted edge cycles must have even lengths.

(iii) The number of occurrences of $e_i = 1/2$ is even. For reasons clearly explained somewhere in [1], the number of twisted edge cycles must be even.

(iv) The number of occurrences of $e_i a_i$ even is even. This is an elementary group theoretic requirement. Observe that $e_i a_i$ even corresponds to an odd permutation of edge pieces. An outer slice turn creates a 4-cycle of edge pieces and a 4-cycle of corner pieces, resulting in an even permutation of cube pieces. Therefore, if the corners don't move, the permutation of the edge pieces must be even.

(v) The sum s has the same parity as r. The proof is straightforward. The sum $s = e_1 a_1 + e_2 a_2 + \cdots + e_r a_r$ has r terms. Remove all the even $e_i a_i$ to leave r' terms and the parity of the sum unchanged. By (iv) the number of terms removed is even and therefore $r' \equiv r \pmod 2$. The remaining terms are the odd a_i and the $a_j/2$ with $a_j \equiv 2 \pmod 4$, all of which are odd. Therefore

$$s \equiv \sum_{i=1, \ a_i \ \text{odd}}^{r} a_i + \sum_{j=1, \ a_j \equiv 2 \pmod 4}^{r} \frac{1}{2} a_j \equiv r' \equiv r \pmod 2.$$

Consider the cube as fixed in space and in its identity state. Then a non-trivial inner slice turn leaves the corner pieces fixed and moves only edge

pieces and face centre pieces. Moreover the action of inner slice turns on edge pieces is commutative. For example, $[f_*, r_*]$ does the permutation (F L U)(B R D) to give the pretty 6-spot pattern; it fixes the edges. As a consequence, the result of $[\mathcal{X}, \mathcal{Y}]$ as defined in (1) does not depend on the order of the moves in $\mathcal{Y}$. Hence we may assume that $\mathcal{Y} = f_*^a r_*^b u_*^c$ for some $a, b, c \in \{0, 1, 2, 3\}$.

When we solve (2) subject to the conditions (i)–(v) we find 301 distinct (non-vacuous) cycle patterns, which we tabulate by number of non-fixed edge pieces.

s	2	3	4	5	6	7	8	9	10	11	12	
	1	1	4	3	9	10	22	27	49	65	110	301

The first few patterns are
$(2; \tilde{2}, \tilde{2})$, $(3; 3)$,
$(4; 2, 2)$, $(4; \tilde{2}, \tilde{6})$, $(4; \tilde{4}, \tilde{4})$, $(4; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2})$,
$(5; 5)$, $(5; \tilde{2}, \tilde{2}, 3)$, $(5; \tilde{2}, 2, \tilde{4})$,
$(6; 2, 4)$, $(6; \tilde{2}, \tilde{10})$, $(6; 3, 3)$, $(6; \tilde{4}, \tilde{8})$, $(6; \tilde{6}, \tilde{6})$,
$(6; \tilde{2}, \tilde{2}, 2, 2)$, $(6; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{6})$, $(6; \tilde{2}, \tilde{2}, \tilde{4}, \tilde{4})$, $(6; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2})$,
$(7; 7)$, $(7; 2, 2, 3)$, $(7; \tilde{2}, \tilde{2}, 5)$, $(7; \tilde{2}, 2, \tilde{8})$, $(7; \tilde{2}, 3, \tilde{6})$, $(7; \tilde{2}, \tilde{4}, 4)$,
$(7; 2, \tilde{4}, \tilde{6})$, $(7; 3, \tilde{4}, \tilde{4})$, $(7; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, 3)$, $(7; \tilde{2}, \tilde{2}, \tilde{2}, 2, \tilde{4})$, $\ldots$,
and the last four are
$(12; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, 2, 2)$, $(12; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{6})$,
$(12; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{4}, \tilde{4})$, $(12; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2})$.

Only even numbers can have hats. The number of hats is even as is also the number of even unhatted numbers plus the number of hatted multiples of 4.

Eight patterns occur as twins, which are identical except for the distribution of the hats:

$$(11; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{4}, 6), \qquad (11; 2, 2, 2, \tilde{4}, \tilde{6});$$
$$(12; \tilde{2}, 4, 4, \tilde{6}), \qquad (12; 2, \tilde{4}, \tilde{4}, 6);$$
$$(12; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, 2, 6), \quad (12; \tilde{2}, 2, 2, 2, 2, \tilde{6});$$
$$(12; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, 4, 4), \quad (12; 2, 2, 2, 2, \tilde{4}, \tilde{4}).$$

It is of course highly desirable to find short edge processes the form $[\mathcal{X}, \mathcal{Y}]$ as in (1) to achieve each one of the 301 cycle patterns, the shorter the better. At present I can do 300. If we restrict processes to my target of 18 moves, the

number is reduced to 295. The six where I have been not totally successful are as follows.

(i) $(10; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2})$. A 10-flip. The best I can do at present is 34 moves,
$$[BRLBR^2DRL'FL'F'UR'L^2, b_*l_*u_*].$$
It is not the most efficient way to flip 10 edges. For example, with the 8-flip $(LRFBUD)^2$ followed by the 4-flip $(D'f_*)^4$ you can flip all edges except {FD, LD} in only 18 moves.

(ii) $(11; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, 2, \tilde{4})$. Current best: 20 moves.

(iii) $(12; \tilde{4}, \tilde{4}, \tilde{4}, \tilde{4}, \tilde{4}, \tilde{4})$. NOT YET. This one is a bit of a mystery. I can think of no obvious reason why the required edge process does not exist. However, all the procedures for obtaining the other 300 cycle patterns have so far failed to yield this one. So it remains a challenge. Find an edge process $[\mathcal{X}, \mathcal{Y}]$ as in (1), or prove that no such process exists. The pattern can be obtained by other means; for example, $([LFL, f_*]Bb_*F'Rr_*L')^3$ (24 moves).

(iv) $(12; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{6})$. Current best: 20 moves.

(v) $(12; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{4}, \tilde{4})$. Current best: 26 moves.

(vi) $(12; \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2}, \tilde{2})$. This is the 12-flip, also known as the superflip. The best I can do at present is 54 moves. This is achieved with a 6-flip of 24 face turns commutated with 3 inner slice turns:
$$[R'D'LF'UR'U'F'BU'DBLF'UR'BLD'UB'FD'U, u_*^2r_*^2b_*].$$
There are of course more efficient ways to achieve a 12-flip. A particularly simple sequence is $(Rf_*)^4(Dl_*)^4(Bu_*)^4$ (24 moves), which is even easier to remember if you are doing it on a real cube,
$$\left((Rf_*)^4 \text{ (rotate the cube 120° about the UFL–DBR diagonal)}\right)^3.$$
However, what we want is a sequence that conforms to the commutator pattern $[\mathcal{X}, \mathcal{Y}]$. Recall that the 12-flip generates the centre of the Rubik cube group.

We finish with a catalogue of the 301 edge process cycle patterns together with examples of sequences of moves that achieve them.

2 edge pieces, 2 cycles

$[FU'RF'U, l_*]$ (12 moves): (FU)+(BU)+ 2e 2e.

3 edge pieces, 1 cycle

$[U^2, f_*]$ (4 moves): (LU RU LD) 3e.

4 edge pieces, 2 cycles

$[U^2, f_*^2]$ (4 moves): (LU RU)(LD RD) 2e 2e.

$[RFR'D, b_*]$ (10 moves): (BD DL LU)+(RD)+ 6e 2e.

$[L'B'L', b_*]$ (8 moves): (BD DL)+(LU UR)+ 4e 4e.

4 edge pieces, 4 cycles

$[D'R^2FB'U^2L, f_*^2]$ (14 moves): (LU)+(LD)+(RU)+(RD)+ 2e 2e 2e 2e.

5 edge pieces, 1 cycle

$[U, f_*]$ (4 moves): (FU UB UR LU DL) 5e.

5 edge pieces, 3 cycles

$[L'UB'U', f_*^2]$ (10 moves): (BL DL UR)(LU)+(RD)+ 3e 2e 2e.

$[L'B'UL', b_*]$ (10 moves): (BD DL)+(LU)+(RU DR) 4e 2e 2e.

6 edge pieces, 2 cycles

$[UR^2, f_*^2]$ (6 moves): (FU BU)(LU RD RU LD) 2e 4e.

$[R^2D'FL', b_*]$ (10 moves): (BL BD LD UL RU)+(RD)+ 10e 2e.

$[U, f_*^2]$ (4 moves): (FU LU RD)(BU RU LD) 3e 3e.

$[UR^2, f_*]$ (6 moves): (FU RU UL LD)+(BU RD)+ 8e 4e.

$[DBR, b_*]$ (8 moves): (FR UR RD)+(BD DL LU)+ 6e 6e.

6 edge pieces, 4 cycles

$[DFL, f_*^2]$ (8 moves): (FD LB)(LU RD)(LD)+(RU)+ 2e 2e 2e 2e.

$[L'U'FL'U'B, u_*r_*^2]$ (16 moves): (FU)+(FL)+(FR LB BD)+(BR)+ 2e 2e 6e 2e.

$[U^2F^2BU^2, r_*^2u_*]$ (12 moves): (FU)+(FL BR)+(FR DB)+(BL)+ 2e 4e 4e 2e.

6 edge pieces, 6 cycles

$[UL'U'F'R'D'R, u_*^2 r_*^2]$ (18 moves): (FL)+(FR)+(FD)+(BU)+(BR)+(BL)+ 2e 2e 2e 2e 2e 2e.

7 edge pieces, 1 cycle

$[UR, f_*]$ (6 moves): (FU FR RB RD UR LU DL) 7e.

7 edge pieces, 3 cycles

$[UR, f_*^2]$ (6 moves): (FU RB)(FR UR DL)(LU RD) 2e 3e 2e.

$[D'F'R'F, r_* f_*^2]$ (12 moves): (FD RB DB BU UR)(LU)+(RD)+ 5e 2e 2e.

$[FR'URF, f_* u_*]$ (14 moves): (FL LU DL RF)+(BU BL)(BR)+ 8e 2e 2e.

$[RD'FD, b_* l_*]$ (12 moves): (FU UB FL)(FR UR RD)+(LD)+ 3e 6e 2e.

$[UR^2, l_* b_*^2]$ (8 moves): (FU DB RD RU)(BU)+(LU DL)+ 4e 2e 4e.

$[LBL'D^2, u_*^2 r_*]$ (12 moves): (FU LF BU)+(FD LD)+(BR BD) 6e 4e 2e.

$[L'B^2L', l_* b_*]$ (10 moves): (FD BD BU)(BR DL)+(LU UR)+ 3e 4e 4e.

7 edge pieces, 5 cycles

$[D'B'D'L'B, u_*^2 f_*^2]$ (14 moves): (FL)+(FD UL DR)(BR)+(LD)+(RU)+ 2e 3e 2e 2e 2e.

$[R'L^2F'BU, f_* r_*^2 u_*^2]$ (16 moves): (FL)+(FR DL)+(BR)+(BL DR)(LU)+ 2e 4e 2e 2e 2e.

8 edge pieces, 2 cycles

$[U^2F, r_* u_*^2]$ (8 moves): (FU BU FR RB BD LB)(FL FD) 6e 2e.

$[B^2D^2L, f_*^2 u_*]$ (10 moves): (FL DL LB RB RU FR RD)+(LU)+ 14e 2e.

$[U^2D, r_* f_*^2]$ (8 moves): (FU UL BU FD LD)(BD RD UR) 5e 3e.

$[UD, f_*]$ (6 moves): (FU UB FD DB)(LU DL RD UR) 4e 4e.

$[B^2RD^2, f_*^2 u_*]$ (10 moves): (FL BL RB DL DR FR)+(LU RU)+ 12e 4e.

$[UB', f_* r_*]$ (8 moves): (FD LU DL)+(BU RU LB BR BD)+ 6e 10e.

$[UFU, u_* f_*^2]$ (10 moves): (FL LU RU DF)+(FR RB LD RD)+ 8e 8e.

8 edge pieces, 4 cycles

$[UD, f_*^2]$ (6 moves): (FU FD)(BU BD)(LU RD)(LD RU) 2e 2e 2e 2e.

$[F^2LR', b_*^2d_*']$ (10 moves): (FL)+(FR LB)(BR)+(LU UR RD DL) 2e 2e 2e 4e.

$[DR'LB, u_*b_*]$ (12 moves): (FL BR)(FR BL)(BU LD UL)+(RD)+ 2e 2e 6e 2e.

$[F'D^2R'U'R', r_*u_*^2]$ (14 moves): (FU DB LU FL BU)+(FR)+(FD)+(BL)+ 10e 2e 2e 2e.

$[U'RB'R', d_*f_*^2]$ (12 moves): (BU LU RD)(BR DB LB)(LD)+(RU)+ 3e 3e 2e 2e.

$[U'LF'UL, f_*u_*]$ (14 moves): (FL BL)+(FR)+(FD RB)(LU DR LD) 4e 2e 2e 3e.

$[RU^2R, l_*^2d_*]$ (10 moves): (FU BU)(FL FR)+(FD BD)(BR LU)+ 2e 4e 2e 4e.

$[DL'DF'R, b_*d_*]$ (14 moves): (FU UL)+(FL DL DR FR)+(BR)+(BL)+ 4e 8e 2e 2e.

$[BR^2F, l_*u_*^2]$ (10 moves): (FU DB FR)+(FL FD BL)+(BU)+(BR)+ 6e 6e 2e 2e.

$[RL^2U^2, f_*^2d_*]$ (10 moves): (FL BR)+(FR)+(BL LU RU)+(LD RD) 4e 2e 6e 4e.

$[UFD, r_*f_*]$ (10 moves): (FL LD)+(FR BU)+(FD DB)+(LU RU)+ 4e 4e 4e 4e.

8 edge pieces, 6 cycles

$[R'L', d_*f_*^2]$ (8 moves): (FL)+(FR LB)(BR)+(LU UR)(LD)+(RD)+ 2e 2e 2e 2e 2e 2e.

$[L'BD^2L'DLF', l_*u_*]$ (18 moves): (FU)+(FL RF DL)+(FD)+(BU)+(BL)+(BD)+ 2e 6e 2e 2e 2e 2e.

$[B'L^2U'B'L'U, u_*^2f_*^2]$ (16 moves): (FL)+(FR)+(BR)+(BL)+(LU DL)+(RU DR)+ 2e 2e 2e 2e 4e 4e.

8 edge pieces, 8 cycles

$[R'U'D'B'UD', f_*^2u_*^2]$ (16 moves): (FL)+(FR)+(BR)+(BL)+(LU)+(LD)+(RU)+(RD)+ 2e 2e 2e 2e 2e 2e 2e 2e.

9 edge pieces, 1 cycle

$[RB, f_*u_*^2]$ (8 moves): (FL BD BR DR BU RU UL LB RF) 9e.

9 edge pieces, 3 cycles

$[U^2B, u_*f_*^2]$ (8 moves): (FU DB RB BL LF)(LU RU)(LD RD) 5e 2e 2e.

$[ULDL, f_*l_*^2]$ (12 moves): (FU RD UL FL LD DF BD)(BU)+(RU)+ 7e 2e 2e.

$[B^2RB'R^2, r_*u_*^2]$ (12 moves): (FU LF RD RB UR BD)+(FR BL)(BU)+ 12e 2e 2e.

$[R^2B, b_*u_*^2]$ (8 moves): (FL BD BR BU)(FR BL)(LD RD RU) 4e 2e 3e.

$[B'RB^2, f_*u_*^2]$ (10 moves): (FL BD BR)(FR UB RD UL LB)+(RU)+ 3e 10e 2e.

$[FD^2F'L, d_*f_*^2]$ (12 moves): (FL DL RB UL DB RF)(BL UR)+(RD)+ 6e 4e 2e.

$[RD'LB, b_*u_*^2]$ (12 moves): (FL BD BL RB)+(FR)+(BU LD RU DR) 8e 2e 4e.

$[U^2L'D, u_*b_*]$ (10 moves): (FL BD DF UR BL)+(FR UL)(LD RD)+ 10e 2e 4e.

$[D'LB'R', u_*b_*]$ (12 moves): (FL BR LB RF BU)(BD RD DL)+(LU)+ 5e 6e 2e.

$[FUF^2, d_*l_*^2]$ (10 moves): (FU UB DF RF)+(FL LB BD)+(LU RU) 8e 6e 2e.

$[DB^2, d_*f_*^2]$ (8 moves): (FR BR BL)(FD RD LU)(BD LD RU) 3e 3e 3e.

$[R'D^2, l_*b_*]$ (8 moves): (FU FD BD)(FR UL)+(BR UR RD DL)+ 3e 4e 8e.

$[L'BLD', f_*u_*]$ (12 moves): (FD BU UL)+(BR LB BD)(LD DR RU)+ 6e 3e 6e.

$[FRF, u_*^2l_*]$ (10 moves): (FU UB)+(FL FR UR BL BR)(FD RD)+ 4e 5e 4e.

$[D^2L'D', d_*b_*]$ (10 moves): (FL BD DF DR)(BR UR BL)+(LU LD)+ 4e 6e 4e.

9 edge pieces, 5 cycles

$[D^2R'U^2, r_*d_*f_*^2]$ (12 moves): (FU BD)(FL LD RB)(FR)+(FD BU)(RU)+ 2e 3e 2e 2e 2e.

$[LU'B'LD, l_*f_*]$ (14 moves): (FD RB)(BU DB)(BL DL)(LU RU)+(RD)+ 2e 2e 2e 4e 2e.

$[U'F'U'R', f_*^2u_*^2]$ (12 moves): (FL BR UR FD DL)(FR)+(BL)+(LU)+(RD)+ 5e 2e 2e 2e 2e.

$[R'L'F'D^2F', f_*u_*^2]$ (14 moves): (FL)+(FR RB)(BL)+(BD LD DR LU)+(RU)+ 2e 2e 2e 8e 2e.

$[RF'RD'F^2, b_*r_*]$ (14 moves): (FU DF RU)(FL BU LU)+(BD)+(LD)+(RD)+ 3e 6e 2e 2e 2e.

$[FRFU, b_*r_*^2]$ (12 moves): (FU)+(FD LU LD BR)(BU)+(BD)+(RU DR)+ 2e 4e 2e 2e 4e.

$[BURD', b_*r_*u_*^2]$ (14 moves): (FU BD DR)+(FL)+(FR DF)(BU LD)+(BR)+ 6e 2e 2e 4e 2e.

$[FRFDB, l_*f_*^2]$ (14 moves): (FU FD UR)(BU DB)+(BL LD)+(LU)+(RD)+ 3e 4e 4e 2e 2e.

$[RD'LU'F, b_*r_*]$ (14 moves): (FL UB)+(FR UL)(FD DB)+(LD)+(RU DR)+ 4e 2e 4e 2e 4e.

9 edge pieces, 7 cycles

$[B'L'B'U'F', f_*^2r_*^2]$ (14 moves): (FU)+(FR FD
BU)(BD)+(LU)+(LD)+(RU)+(RD)+ 2e 3e 2e 2e 2e 2e 2e.

$[L^2DB'RU'F', b_*l_*u_*]$ (18 moves): (FU FR)(FL
FD)+(BU)+(BR)+(BD)+(RU)+(RD)+ 2e 4e 2e 2e 2e 2e 2e.

10 edge pieces, 2 cycles

$[R^2L'D, u_*b_*]$ (10 moves): (FL BD DF DL UR RD RF LB)(BR UL) 8e 2e.

$[BD^2L, u_*r_*^2]$ (10 moves): (FU BU FD BD BR LF FR LU DR)+(BL)+ 18e 2e.

$[BD', f_*u_*^2]$ (8 moves): (FL BD BR)(FR BU BL DF DL RD UR) 3e 7e.

$[RB, u_*^2f_*^2]$ (8 moves): (FL BD BR DR BU UL)(FR UR DL BL) 6e 4e.

$[BRD, b_*u_*^2]$ (10 moves): (FL RD UB DL LU RF RU RB)+(FD BL)+ 16e 4e.

$[R'BD^2, u_*^2f_*^2]$ (10 moves): (FL BD BR BL FR)(BU DL DR UL UR) 5e 5e.

$[U^2LD, l_*^2b_*]$ (10 moves): (FU LF BD UB LD UL LB)+(FD RU DR)+ 14e 6e.

$[B^2RU, b_*^2r_*]$ (10 moves): (FU LB DF RF DR UL)+(BU BD RU LD)+ 12e 8e.

$[RD, l_*^2b_*]$ (8 moves): (FU RB BD UB LU)+(FR UR RD DL DF)+ 10e 10e.

10 edge pieces, 4 cycles

$[R^2B, f_*^2u_*^2]$ (8 moves): (FL BD BR BU)(FR BL)(LU LD)(RU RD) 4e 2e 2e 2e.

$[BLR, b_*d_*^2]$ (10 moves): (FL DB BL FR UB BR)(LU DR)(LD)+(RU)+ 6e 2e 2e
2e.

$[U^2RB^2LD, f_*u_*^2]$ (14 moves): (FU LD UR FR FD)+(FL BR)(BL LU)(RD)+
10e 2e 2e 2e.

$[LD^2FRU', f_*^2d_*]$ (14 moves): (FL LD BD BL RB BU FR)+(LU)+(RU)+(RD)+
14e 2e 2e 2e.

$[BD^2, f_*^2u_*^2]$ (8 moves): (FL BD BR)(FR BU BL)(LU RU)(LD RD) 3e 3e 2e 2e.

$[U'RU^2B, f_*^2u_*^2]$ (12 moves): (FL BD BR BL FR)(BU RU LD)(LU)+(RD)+ 5e
3e 2e 2e.

$[R'D'LU^2, b_*u_*^2]$ (12 moves): (FL BD FR BL)+(FD DR RU)(BR)+(LU DL) 8e
3e 2e 2e.

$[D'R'B', f_*^2u_*^2]$ (10 moves): (FL BU BR DF)(FR DR UL BL)(LD)+(RU)+ 4e 4e
2e 2e.

$[L'FR'D, b_*d_*]$ (12 moves): (FL BD)(FR)+(FD DR UR DL LU)(BR BL)+ 2e 2e
5e 4e.

$[LU^2RD', u_*^2f_*]$ (12 moves): (FL BR)(FR BD DF UL)(BL LD UR)+(RD)+ 2e 4e 6e 2e.

$[F^2D, r_*u_*^2]$ (8 moves): (FU DF)+(FL FR)(BU DR LD BD)+(BR BL) 4e 2e 8e 2e.

$[RDURB, r_*u_*^2]$ (14 moves): (FU DF LU BD UB RF)+(FL)+(BR)+(BL DL)+ 12e 2e 2e 4e.

$[DRUR, u_*l_*^2]$ (12 moves): (FU BD)(FR DR RB)+(FD BU LB)+(LU LD) 2e 6e 6e 2e.

$[UFR^2B, l_*u_*^2]$ (12 moves): (FU UB BD)+(FL LB RU RB UL)+(FR)+(FD)+ 6e 10e 2e 2e.

$[BUF, r_*d_*^2]$ (10 moves): (FU DF BU RU)+(FL UL FR RB)+(BL)+(BD)+ 8e 8e 2e 2e.

$[D'LF^2U^2, f_*^2u_*]$ (12 moves): (FL)+(FR FD LB)(BR RD LU)+(BD LD RU) 2e 3e 6e 3e.

$[F^2R', f_*r_*u_*^2]$ (10 moves): (FU BD FD)(FL DR BL LU)(FR RU)+(BR)+ 3e 4e 4e 2e.

$[UBLD^2, d_*l_*]$ (12 moves): (FU LF BL)(FR FD)(BU DB DL)+(BR UR)+ 3e 2e 6e 4e.

$[FRL', f_*r_*u_*^2]$ (12 moves): (FU LF)+(FR RB)(BL BD)+(LU UR RD DL) 4e 2e 4e 4e.

$[RBLDB', u_*^2f_*]$ (14 moves): (FL BR UR LD DR)+(FR FD)+(BU LB)+(LU)+ 10e 4e 4e 2e.

$[B'R^2F^2R'U^2, u_*f_*^2]$ (14 moves): (FL RB BL)+(FR)+(BU DB DL DR)+(LU RU)+ 6e 2e 8e 4e.

$[UB'U'L'D', r_*u_*]$ (14 moves): (FL RF DL)+(FD DB DR)+(BU)+(BR LB UL)+ 6e 6e 2e 6e.

$[ULFD', f_*u_*^2]$ (12 moves): (FL BR FD)(BU DB BL)(LU DL)+(RU RD)+ 3e 3e 4e 4e.

$[BDRUL, b_*l_*]$ (14 moves): (FU UB FL FD)+(FR UL)+(BR DB)+(RU DR)+ 8e 4e 4e 4e.

$[R'B', l_*f_*u_*^2]$ (10 moves): (FL DB UL)+(FR DR)+(FD UR)+(BU BR LB)+ 6e 4e 4e 6e.

10 edge pieces, 6 cycles

$[FRLU, f_*^2u_*^2]$ (12 moves): (FL)+(FR BL)(FD UB)(BR)+(LU LD)(RU RD) 2e

2e 2e 2e 2e 2e.

$[L'UD, u_*r_*^2f_*]$ (12 moves): (FU DF)(FL)+(FR DR LU LB)(BU)+(BD)+(RU)+ 2e 2e 4e 2e 2e 2e.

$[UDB, l_*b_*^2]$ (10 moves): (FU)+(FD UB)(BR DL BL)+(BD)+(LU RD)(RU)+ 2e 2e 6e 2e 2e 2e.

$[L'FD'RB'R', r_*u_*f_*]$ (18 moves): (FU BD BR FD LF)+(FR)+(BU)+(BL)+(RU)+(RD)+ 10e 2e 2e 2e 2e 2e.

$[DRUDF, r_*^2u_*^2]$ (14 moves): (FU BD UR)(FL)+(FR)+(FD UL BU)(BR)+(BL)+ 3e 2e 2e 3e 2e 2e.

$[RLD'F^2, r_*^2u_*^2b_*]$ (14 moves): (FU FD)+(FR)+(BU BD BL)(LU)+(LD UR)(RD)+ 4e 2e 3e 2e 2e 2e.

$[FDUL, l_*^2f_*]$ (12 moves): (FU BD)(FR BU)+(FD LB)+(LU DR)(LD)+(RU)+ 2e 4e 4e 2e 2e 2e.

$[ULB^2D^2R'F', b_*r_*^2]$ (16 moves): (FU)+(FR UL RU DR)+(FD)+(BU)+(BL DL)+(BD)+ 2e 8e 2e 2e 4e 2e.

$[F'UL'B'U', l_*u_*^2b_*]$ (16 moves): (FU BR BD)+(FL)+(FR)+(FD)+(BU)+(BL DL UR)+ 6e 2e 2e 2e 2e 6e.

$[RU'FDL', r_*d_*f_*]$ (16 moves): (FU UR)+(FL)+(FD)+(BR DR)+(BL BD DL)+(LU)+ 4e 2e 2e 4e 6e 2e.

$[B'U'R'L'U'L, d_*r_*^2]$ (16 moves): (FU DR)+(FL FR)+(FD)+(BU)+(BR BL)+(BD RU)+ 4e 4e 2e 2e 4e 4e.

10 edge pieces, 8 cycles

$[D'FBU^2R, u_*^2f_*^2r_*]$ (16 moves): (FU BD)(FL RF)(BR)+(BL)+(LU)+(LD)+(RU)+(RD)+ 2e 2e 2e 2e 2e 2e 2e 2e.

$[FRB'L^2F'D', d_*r_*^2f_*^2]$ (18 moves): (FU)+(FR DF LB)+(BU)+(BD)+(LU)+(LD)+(RU)+(RD)+ 2e 6e 2e 2e 2e 2e 2e 2e.

$[U'FL'DB', b_*u_*l_*]$ (16 moves): (FU)+(FL)+(FR BU)+(FD)+(BR DB)+(BL)+(LU)+(LD)+ 2e 2e 4e 2e 4e 2e 2e 2e.

10 edge pieces, 10 cycles

$[BRLBR^2DRL'FL'F'UR'L^2, b_*l_*u_*]$ (34 moves): (FL)+(FR)+(FD)+(BU)+(BR)+(BL)+(LU)+(LD)+(RU)+(RD)+ 2e 2e 2e 2e 2e 2e 2e 2e 2e 2e.

11 edge pieces, 1 cycle

$[R'BD, f_*u_*^2]$ (10 moves): (FL BD BR FD RF LB DL RD UR BU LU) 11e.

11 edge pieces, 3 cycles

$[URD, b_*^2d_*]$ (10 moves): (FU FD)(FL LU RD UB BR RF BD)(LD RU) 2e 7e 2e.

$[DBR, r_*f_*^2]$ (10 moves): (FU RB BL UR DL DF BD RF UB)(LU)+(RD)+ 9e 2e 2e.

$[LBUD, l_*^2b_*]$ (12 moves): (FU UL RU DL UB DF BR DB)+(FL BL)(RD)+ 16e 2e 2e.

$[RBD', f_*^2u_*^2]$ (10 moves): (FL BD BR)(FR DF BL RD UB LU)(LD RU) 3e 6e 2e.

$[L'DBR^2, b_*d_*]$ (12 moves): (FL BD RD)(FR UB LU UR FD LB BR)+(LD)+ 3e 14e 2e.

$[R^2BD, f_*^2u_*^2]$ (10 moves): (FL BD BR BU)(FR BL LU FD RD)(LD RU) 4e 5e 2e.

$[UL'DB, u_*f_*]$ (12 moves): (FL BU DF BD RF RD BR LB)(LU RU)+(LD)+ 8e 4e 2e.

$[BU'DR, l_*u_*]$ (12 moves): (FL BD DF FR RB BL)+(BU)+(LU DL RD UR) 12e 2e 4e.

$[BDR^2, d_*^2b_*]$ (10 moves): (FL BL RD DL LU RF RB)+(FD RU)+(BU BD) 14e 4e 2e.

$[U^2BLU', l_*b_*]$ (12 moves): (FU)+(FL DL LU UB BR)(FD LB UR RD BD)+ 2e 5e 10e.

$[L'DB', u_*b_*]$ (10 moves): (FL BD DF DR BU BR LB)(FR UL RU)+(LD)+ 7e 6e 2e.

$[L^2F'L, u_*^2f_*^2l_*]$ (12 moves): (FU DL FL UL)+(FR)+(FD DR BR UR UB BL) 8e 2e 6e.

$[FDR, u_*b_*^2]$ (10 moves): (FU DB)(FL RF DF)+(BR DR UL LB DL UR)+ 2e 6e 12e.

$[BDR, l_*^2f_*]$ (10 moves): (FU UR LU FR)+(FD BU)(BR DR DB LB DL)+ 8e 2e 10e.

$[BD, f_*^2u_*^2]$ (8 moves): (FL BD BR)(FR BU BL LD RU)(FD RD LU) 3e 5e 3e.

$[R'BD', f_*^2u_*^2]$ (10 moves): (FL BD BR DF)(FR BL RD LU)(BU DL UR) 4e 4e 3e.

15

$[FRD, r_*f_*^2]$ (10 moves): (FU BU BR RU LD LF)+(FR RD LU)(FD DB)+ 12e 3e 4e.

$[FRU, d_*l_*]$ (10 moves): (FU LF BL)(FR FD RD UR BR)+(BU DB LU)+ 3e 10e 6e.

$[RB'D, u_*^2f_*]$ (10 moves): (FL BU BR)(FR FD UL LB)+(BD RU DL RD)+ 3e 8e 8e.

$[U'F^2R', l_*d_*]$ (10 moves): (FU DR BR RF BU DB RU)(FL LB)+(FD UL)+ 7e 4e 4e.

$[R^2BD, b_*u_*^2]$ (10 moves): (FL BD BR BU)(FR BL DF DL LU)+(RU DR)+ 4e 10e 4e.

$[R^2B'D, b_*u_*]$ (10 moves): (FL DB BU BL)+(FR DF DL LU BR)(RU DR)+ 8e 5e 4e.

$[FR', f_*r_*^2u_*^2]$ (10 moves): (FU DR BU BL LU FL)(FR FD RU)+(BR DB)+ 6e 6e 4e.

$[LDFD^2, l_*u_*]$ (12 moves): (FU RF DB FD)+(FL DL BL)+(BU LU RD BR) 8e 6e 4e.

$[R'B'D', b_*u_*]$ (10 moves): (FL DB BU DR LB)(FR RB RU)+(FD DL LU)+ 5e 6e 6e.

11 edge pieces, 5 cycles

$[F^2LR, l_*b_*^2]$ (10 moves): (FU FD BU)(FL FR)(BR BL)(LU RD)(LD RU) 3e 2e 2e 2e 2e.

$[LFUR, b_*d_*^2]$ (12 moves): (FU RU DR LU RB)(FL)+(FR BL)(FD UB)(LD)+ 5e 2e 2e 2e 2e.

$[F'UL'B, l_*d_*b_*]$ (14 moves): (FU BR RD BD RU BL DL)(FL)+(FR)+(BU)+(LU)+ 7e 2e 2e 2e 2e.

$[DUF, u_*b_*^2]$ (10 moves): (FU FD)(FL RF BR DL)+(BU BD)(LU RD)(RU)+ 2e 8e 2e 2e 2e.

$[R'L'FU^2, l_*u_*f_*^2]$ (14 moves): (FL)+(FR LB)(FD RD UB DL BD RU)+(BR)+(LU)+ 2e 2e 12e 2e 2e.

$[LBD, f_*^2u_*^2]$ (10 moves): (FL BR)(FR BU BL DF)(BD DR UL)(LD)+(RU)+ 2e 4e 3e 2e 2e.

$[U'L'D^2B, f_*^2l_*]$ (12 moves): (FU LU FD)+(FL BL)(BU BR DR)(BD)+(LD RU) 6e 2e 3e 2e 2e.

$[RURUB, r_*^2f_*d_*]$ (16 moves): (FU BL UR DR BR)+(FL FR

16

DL)(FD)+(BU)+(BD)+ 10e 3e 2e 2e 2e.

$[U^2R', f_*^2 r_*^2 d_*]$ (10 moves): (FU BU)(FL RD RB RU)(FR DL)+(FD BD)(LU)+ 2e 4e 4e 2e 2e.

Normal 6-cycle
$[U'F'RD'F', u_* r_* f_*^2]$ (16 moves): (FU BL DF RF LF DB)(BU)+(BR)+(LD DR)+(RU)+ 6e 2e 2e 4e 2e.

Twisted 6-cycle
$[BD^2LF, r_*^2 b_*]$ (12 moves): (FU BD)(FL UL BU)+(FR BR)(FD RU)(LD RD)+ 2e 6e 2e 2e 4e.

$[DB'LU, r_* d_* f_*^2]$ (14 moves): (FU DB RD RF)+(FL BR UR UL)(FD)+(BU)+(LD)+ 8e 4e 2e 2e 2e.

$[BR'L', l_* u_*^2 b_*]$ (12 moves): (FL BD RB UB BL)+(FR FD)+(LU DR)(LD)+(RU)+ 10e 4e 2e 2e 2e.

$[FD'L'D^2R', u_*^2 f_*]$ (14 moves): (FU BD FD LF RB)(FR)+(BL)+(LU)+(LD DR RU)+ 5e 2e 2e 2e 6e.

$[R'UF'L', u_*^2 b_*]$ (12 moves): (FU DF)(FL BR FR BU)+(BL)+(LU UR RD)+(LD)+ 2e 8e 2e 6e 2e.

$[F^2L'U'B', f_*^2 u_*^2]$ (12 moves): (FL BD BR)(FR BL DF)(BU LU RD)(LD)+(RU)+ 3e 3e 3e 2e 2e.

$[D'BU^2F, b_* u_*^2]$ (12 moves): (FL BR BU)(FR BL)(FD DB UR)(LU RD)+(LD)+ 3e 2e 3e 4e 2e.

$[BLD, f_*^2 u_*^2]$ (10 moves): (FL BR)(FR UR)+(FD UB)(BL LD)+(BD DR UL) 2e 4e 2e 4e 3e.

$[RBLB^2, d_*^2 r_* b_*]$ (14 moves): (FU RF)+(FL)+(BU RB BL DR)+(BD LD UR)(LU)+ 4e 2e 8e 3e 2e.

$[LU^2R, l_* b_* u_*^2]$ (12 moves): (FU BD BU)(FL)+(FR DR LU)+(BR LB UR)+(LD)+ 3e 2e 6e 6e 2e.

$[UFRB, b_*^2 d_*]$ (12 moves): (FU FR LF BL RB)(FD LU)+(BD DR)+(LD)+(RU)+ 5e 4e 4e 2e 2e.

$[F'R^2B'L, r_*^2 b_* u_*]$ (14 moves): (FU DR)+(FL)+(FR BU BL)+(FD RB RU LD)(BD)+ 4e 2e 6e 4e 2e.

$[RFUB, l_* d_*^2]$ (12 moves): (FU LU UB BD)+(FL BR)(FR DR)+(FD)+(BL RU)+ 8e 2e 4e 2e 4e.

$[F^2RU'BL, f_* u_*^2]$ (14 moves): (FL DF)+(FR BR LB)+(BU DB)(LU DL RU)+(RD)+ 4e 6e 2e 6e 2e.

17

$[R'L^2D^2, r_*u_*f_*^2]$ (12 moves): (FL LU RU)+(FR LB)+(FD BU BD)(BR)+(LD RD)+ 6e 4e 3e 2e 4e.

$[L^2UR'DF, f_*d_*r_*]$ (16 moves): (FU RF)+(FL LD)+(FD BU RB BD)(BL RD)+(RU)+ 4e 4e 4e 4e 2e.

$[BR'D'B'L^2, r_*f_*^2u_*^2]$ (16 moves): (FL DL)+(FR DF)+(BU LU)(BR RU)+(BL DB RD)+ 4e 4e 2e 4e 6e.

$[RUFL, f_*r_*^2]$ (12 moves): (FU BD FL)(FD LB)+(BU BR)+(LU RU)+(LD DR)+ 3e 4e 4e 4e 4e.

11 edge pieces, 7 cycles

$[F'B'R^2, r_*^2d_*b_*]$ (12 moves): (FU DF)(FL RB)(FR)+(BU)+(BL)+(BD)+(LD RD RU) 2e 2e 2e 2e 2e 2e 3e.

$[RF'BU, d_*f_*^2r_*]$ (14 moves): (FU BD)(FL)+(FR RD)+(FD BU)(BR LU)(LD)+(RU)+ 2e 2e 4e 2e 2e 2e 2e.

$[UL'U^2D'B'D, l_*f_*^2u_*^2]$ (18 moves): (FU RD DB BU LU)(FL)+(FR)+(BR)+(BL)+(LD)+(RU)+ 5e 2e 2e 2e 2e 2e 2e.

$[R'B'UL'U'D', l_*d_*f_*^2]$ (18 moves): (FU UR BD RB)+(FR FD)(BU)+(BL)+(LU)+(LD)+(RD)+ 8e 2e 2e 2e 2e 2e 2e.

$[D'R'DF'U', u_*^2f_*r_*^2]$ (16 moves): (FU)+(FL)+(FR LD DF)+(BU UL BL)(BR)+(BD)+(RU)+ 2e 2e 6e 3e 2e 2e 2e.

$[URLU'B, b_*u_*^2r_*^2]$ (16 moves): (FU)+(FL)+(FR DL)+(FD)+(BU)+(BR DR LB LU)(BD)+ 2e 2e 4e 2e 2e 4e 2e.

$[B'UD, r_*u_*f_*^2]$ (12 moves): (FU DB)+(FL BU BR)+(FD)+(BL)+(LU)+(LD)+(RU DR) 4e 6e 2e 2e 2e 2e 2e.

$[B'U'D'F, l_*d_*f_*^2]$ (14 moves): (FU)+(FR UR LB)(FD UB)+(BR)+(BD UL)+(LD)+(RD)+ 2e 3e 4e 2e 4e 2e 2e.

$[DRF'DL'B^2, b_*d_*r_*]$ (18 moves): (FU BR)+(FL)+(FR FD)(BL)+(BD)+(LU RD)+(LD UR)+ 4e 2e 2e 2e 2e 4e 4e.

11 edge pieces, 9 cycles

$[R'U'D'B'UD', f_*^2l_*u_*^2]$ (18 moves): (FU FD BD)(FL)+(FR)+(BR)+(BL)+(LU)+(LD)+(RU)+(RD)+ 3e 2e 2e 2e 2e 2e 2e 2e 2e.

$[UB^2RFB'RB', d_*r_*^2f_*^2]$ (20 moves): (FU)+(FL)+(FR LB)+(FD BR)(BD)+(LU)+(LD)+(RU)+(RD)+ 2e 2e 4e 2e 2e 2e 2e 2e 2e.

12 edge pieces, 2 cycles

$[B'L'U'R, f_*r_*^2]$ (12 moves): (FU BD BL FR DF LU DL RD UR UB)(FL RB) 10e 2e.

$[F'L'DB', b_*u_*]$ (12 moves): (FU BD FR RB BL LF DR BU DF LU UR)+(LD)+ 22e 2e.

$[RFL, f_*l_*^2]$ (10 moves): (FU BD FL)(FR RB BU RU UL LD DR FD LB) 3e 9e.

$[RLD, d_*r_*^2]$ (10 moves): (FU FR LF FD BU BL RB BD)(LU DR RU DL) 8e 4e.

$[L'DR'F^2, r_*f_*]$ (12 moves): (FU BL)+(FL RB FR UB UR LU DL RD FD DB)+ 4e 20e.

$[L'DF'R, b_*r_*^2]$ (12 moves): (FU BD BR BL FL)(FR DL FD BU LU UR RD) 5e 7e.

$[FL', f_*^2r_*^2u_*^2]$ (10 moves): (FU DR BU BL DL FL)(FR FD UL BD BR UR) 6e 6e.

$[D'R'FU', b_*d_*]$ (12 moves): (FU BD RB FR LB DL LU UR RD)+(FL UB FD)+ 18e 6e.

$[FUR', b_*l_*u_*]$ (12 moves): (FU BL DL BU)+(FL LU RD BD RU BR RF DF)+ 8e 16e.

$[ULF, f_*d_*r_*]$ (12 moves): (FU DL BD FL BR DR BL)+(FR DF UL UB UR)+ 14e 10e.

$[FU'RD', u_*f_*]$ (12 moves): (FU BD BR LB FR UB)+(FL RU UL LD DR DF)+ 12e 12e.

12 edge pieces, 4 cycles

$[R'L'F', f_*^2r_*^2]$ (10 moves): (FU BD DL FD BU UR)(FL FR)(BR BL)(LU RD) 6e 2e 2e 2e.

$[ULDF, d_*^2l_*]$ (12 moves): (FU BL FR DR LU UB BD DF)(FL)+(BR)+(LD UR) 8e 2e 2e 2e.

$[RFLU, l_*d_*^2]$ (12 moves): (FU DL UB BD FR DR FD)+(FL BR)(BL)+(LU RU) 14e 2e 2e 2e.

$[UFUR, l_*d_*f_*^2]$ (14 moves): (FU BD BU FL FR FD LU RB LB)+(LD)+(RU)+(RD)+ 18e 2e 2e 2e.

$[U'B'D, r_*^2u_*^2]$ (10 moves): (FU BD BL LD FR)(FL RU BR)(FD BU)(LU RD) 5e 3e 2e 2e.

$[URD'BD, d_*r_*^2]$ (14 moves): (FU BD RU)(FL RD LD UL LB BR RF)(FD)+(BU)+ 3e 7e 2e 2e.

$[B'LF'UD, b_*r_*^2]$ (14 moves): (FU BD FL)(FR RB RU DL LU BL)+(FD BU)(RD)+ 3e 12e 2e 2e.

$[L^2UD, b_*u_*^2]$ (10 moves): (FU DB FD UB)(FL BL)(FR BR)(LU UR RD DL) 4e 2e 2e 4e.

$[B'R'LU', u_*r_*^2]$ (12 moves): (FU BD BL LF FR RB)(FD)+(BU)+(LU DL RD UR) 6e 2e 2e 4e.

$[BL'UR'F, u_*b_*]$ (14 moves): (FU BU BR LB FL RF BD)(FD UR)(LU LD)+(RD)+ 7e 2e 4e 2e.

$[R'B^2D', f_*l_*u_*^2]$ (12 moves): (FU BL BR DB UB)+(FL FR DR LU)(FD LD)(RU)+ 10e 4e 2e 2e.

$[FRBD, r_*^2b_*]$ (12 moves): (FU UR UL BL RF DB)+(FL RB)(FD BU)(LD RD)+ 12e 2e 2e 4e.

$[DFUR^2, b_*^2u_*l_*]$ (14 moves): (FU BL BU RF LU DL LF BD)+(FD)+(BR UR)+(RD)+ 16e 2e 4e 2e.

$[RF'D, b_*d_*r_*]$ (12 moves): (FU BD BL FD DR)(FL)+(FR UR BR UL UB)(LD)+ 5e 2e 5e 2e.

$[UFDB, l_*b_*^2]$ (12 moves): (FU FR UB FD)+(FL UR DL BL RB)(BD)+(LU RD) 8e 5e 2e 2e.

$[U^2L^2F'L, r_*f_*^2d_*]$ (14 moves): (FU BU DR LB LF LU)(FR FD)(BR LD DB)+(RU)+ 6e 2e 6e 2e.

$[UFDL^2, r_*^2u_*]$ (12 moves): (FU BD)(FL RD RF BR UR)+(FD BL UB)+(LU LD) 2e 10e 6e 2e.

$[B'L'UR', r_*f_*^2u_*]$ (14 moves): (FU BR BD)+(FL LB RU RF DF LD DR)+(BU)+(LU)+ 6e 14e 2e 2e.

$[BDF'R, f_*r_*^2]$ (12 moves): (FU BD BL LD)+(FL BR)(FR UR LU DR)+(FD BU) 8e 2e 8e 2e.

$[D'B'L', d_*r_*f_*^2]$ (12 moves): (FU BR UL DB LB RF)+(FL BU RD FD)+(LD)+(RU)+ 12e 8e 2e 2e.

$[U'L'D', b_*r_*^2u_*^2]$ (12 moves): (FU BR LU RF FD)+(FL BU LB UR LD)+(BD)+(RD)+ 10e 10e 2e 2e.

$[B'URD, r_*^2d_*]$ (12 moves): (FU BD BL)(FL RD BR RF)(FD BU UR)(LU LD) 3e 4e 3e 2e.

$[F'U'RLF', d_*r_*^2]$ (14 moves): (FU BD DR)(FL)+(FR DL UL LB BR)+(FD BU UR) 3e 2e 10e 3e.

$[FR^2LD^2, f_*^2r_*^2u_*]$ (14 moves): (FU BL BU)(FL RB DR DL DB DF)(FR)+(LU

UR)+ 3e 6e 2e 4e.

$[FR'B^2U, d_*f_*^2r_*]$ (14 moves): (FU BL LU)(FL FD LD UB)+(FR)+(BR RD DB UR) 3e 8e 2e 4e.

$[FD^2B'L, f_*r_*^2]$ (12 moves): (FU BD BL)(FL BR)(FR UL LD BU FD)+(RU RD)+ 3e 2e 10e 4e.

$[BUFR'D, f_*u_*^2]$ (14 moves): (FU BD BL)+(FL BR UB)(FR)+(FD RU UL LD DR) 6e 3e 2e 5e.

$[DRUF, u_*l_*^2]$ (12 moves): (FU BD)(FL RD RF DL)+(FD UL BU)(BR LB UR)+ 2e 8e 3e 6e.

$[DFU^2R, l_*^2f_*u_*]$ (14 moves): (FU BL DL LF BR)(FR)+(FD DR)+(BU UR DB UL) 5e 2e 4e 4e.

Normal 4-cycles, twisted 6-cycle
$[RL'UF^2, r_*^2d_*]$ (12 moves): (FU BD BU FD)(FL)+(FR LB BR)+(LU DR LD UR) 4e 2e 6e 4e.

Twisted 4-cycles, normal 6-cycle
$[D^2L'F'U'B, f_*^2d_*]$ (14 moves): (FU DB FD DL UB UR)(FL LB)+(FR BR)+(LU RD) 6e 4e 4e 2e.

$[L^2UDB, u_*^2f_*]$ (12 moves): (FU UB FD DB)(FL BR)(FR DR)+(BL UR LU DL)+ 4e 2e 4e 8e.

$[R'U'FB, u_*^2b_*l_*]$ (14 moves): (FU BD)+(FL DR RB UR RF BU LU)+(FD)+(BL DL)+ 4e 14e 2e 4e.

$[L'B'RD'F, u_*^2f_*]$ (14 moves): (FU BD)(FL LB RF)+(FD BR)+(BU UL LD DR RU) 2e 6e 4e 5e.

$[BU'D'R, d_*f_*]$ (12 moves): (FU BU BL)+(FL RB)(FR DB DF)+(LU DL RD UR) 6e 2e 6e 4e.

$[RU'F, b_*u_*r_*^2]$ (12 moves): (FU DF LF)+(FR DL BL DR BR BD)+(BU LU)+(RU)+ 6e 12e 4e 2e.

$[LB'D^2, d_*f_*^2r_*^2]$ (12 moves): (FU BU)+(FL RB RU LU)+(FR DF DB DR DL)+(BL)+ 4e 8e 10e 2e.

$[R'DL^2F^2, u_*^2f_*l_*]$ (14 moves): (FU BL BR)+(FL FR DB)+(FD LD BU RD UL)+(RU)+ 6e 6e 10e 2e.

$[U'FDFL', r_*^2f_*u_*]$ (16 moves): (FU BR FR LF)+(FD UR LD BD)+(BU LB DR)+(LU)+ 8e 8e 6e 2e.

$[L'F'BR', r_*^2f_*^2]$ (12 moves): (FU BD BL)(FL BU FD)(FR DR UL)(BR UR DL) 3e 3e 3e 3e.

$[DF'B'U'R^2, d_*l_*]$ (14 moves): (FU BD DF)(FL FR)+(BU DL RD)(BR UR LU BL)+ 3e 4e 3e 8e.

$[R'UDB'U, u_*l_*]$ (14 moves): (FU BD DF)(FL RB BL)+(FR LU RU)+(BU DL DR) 3e 6e 6e 3e.

$[BUL, d_*f_*r_*]$ (12 moves): (FU LF)+(FR BU)+(FD LU BR RU LD)(BL BD DR) 4e 4e 5e 3e.

$[R'LU'B^2, l_*b_*]$ (12 moves): (FU BD BL)+(FL RF BU)(FD BR)+(LU UR RD DL) 6e 3e 4e 4e.

$[B^2UL', f_*u_*r^2_*]$ (12 moves): (FU LF)+(FR DR BL BR)(FD DL BD RU)(BU UL)+ 4e 4e 4e 4e.

$[L'F^2B'U^2, b_*r^2_*d_*]$ (14 moves): (FU BU)+(FL DR BL DL DF DB)+(FR UL)+(BR RU)+ 4e 12e 4e 4e.

$[R'UB, f^2_*r_*d_*]$ (12 moves): (FU FL LD)+(FR FD)+(BU RU)+(BR UL LB DB RD)+ 6e 4e 4e 10e.

$[U'FL', f^2_*r_*d_*]$ (12 moves): (FU BR)+(FL DL DR DB)+(FR BL FD UL)+(BU UR)+ 4e 8e 8e 4e.

$[D'F'B^2, f^2_*u_*r^2_*]$ (12 moves): (FU BL UL)+(FL RU BU RB)+(FR FD LD)+(BD RD)+ 6e 8e 6e 4e.

$[R^2D'LF', l_*u^2_*f_*]$ (14 moves): (FU BL UR)+(FL RF BU)+(FD LU DR)+(BR DL DB)+ 6e 6e 6e 6e.

12 edge pieces, 6 cycles

$[F^2D^2, f^2_*r^2_*u^2_*]$ (10 moves): (FU BU)(FL FR)(FD BD)(BR BL)(LU RU)(LD RD) 2e 2e 2e 2e 2e 2e.

$[RLDU, d_*f^2_*]$ (12 moves): (FU FD)(FL LB BR RF)(BU BD)(LU)+(LD DR)(RU)+ 2e 4e 2e 2e 2e 2e.

Normal 6-cycle
$[U'D'F^2R, r^2_*f_*d_*]$ (14 moves): (FU RB UB BL FL FR)(FD)+(BD)+(LU)+(LD UR)(RD)+ 6e 2e 2e 2e 2e 2e.

Twisted 6-cycle
$[FRD'BR, f_*r^2_*]$ (14 moves): (FU BD)(FL BL)(FR LD)(FD BU)(BR UL UR)+(RD)+ 2e 2e 2e 2e 6e 2e.

$[F'UDL', d_*r_*f^2_*]$ (14 moves): (FU BL)(FL FD FR BU BR)+(BD)+(LU)+(LD)+(RU DR) 2e 10e 2e 2e 2e 2e.

$[F'D'F'L'U, f^2_*u^2_*l_*]$ (16 moves): (FU UR UB FR LD BL

BD)+(FL)+(FD)+(BR)+(LU)+(RD)+ 14e 2e 2e 2e 2e 2e.

$[F'L'D'R, u_*^2 r_*^2]$ (12 moves): (FU BD LU)(FL BR)(FR)+(FD BU RD)(BL)+(LD RU) 3e 2e 2e 3e 2e 2e.

$[RLU^2D', u_*^2 l_*]$ (12 moves): (FU BD DF)(FL UB)+(FR BL)(BR)+(LU RU)(LD RD) 3e 4e 2e 2e 2e 2e.

$[ULUF, r_*^2 f_* u_*^2]$ (14 moves): (FU)+(FL UL UR DL FD)(FR)+(BU RD BR)(BL)+(BD)+ 2e 5e 2e 3e 2e 2e.

$[L^2 BUD'R, b_* d_* r_*^2]$ (16 moves): (FU UB RB DF)+(FL FR BD)(BL)+(LU DR)(LD)+(RU)+ 8e 3e 2e 2e 2e 2e.

Normal 4-cycles
$[F'D'R'F', u_*^2 f_*^2 r_*^2]$ (14 moves): (FU FD DR RF)(FL)+(BU BD LB UL)(BR)+(LD)+(RU)+ 4e 2e 4e 2e 2e 2e.

Twisted 4-cycles
$[D'RLB'L, b_* d_* r_*]$ (16 moves): (FU DB)(FL BR)(FR LU)+(FD RD)(BU BL)+(LD UR) 2e 2e 4e 2e 4e 2e.

$[L^2 B'U'D', d_* l_* f_*^2]$ (14 moves): (FU)+(FL RF DB RB BL)(FD UB)+(LU DL)(RU)+(RD)+ 2e 5e 4e 2e 2e 2e.

$[DRL, u_*^2 l_*^2 f_*]$ (12 moves): (FU UR LD FD)(FL RF)(BU DR DB)+(BR)+(BL)+(LU)+ 4e 2e 6e 2e 2e 2e.

$[U'DL^2B', f_*^2 l_* d_*]$ (14 moves): (FU DB)(FL DR RB LB)+(FR DL)+(FD)+(BU)+(LU UR) 2e 8e 4e 2e 2e 2e.

$[UR'BD'L^2, f_* d_* r_*]$ (16 moves): (FU BD BL FD RF LU)+(FL LD)+(BU)+(BR)+(RU)+(RD)+ 12e 4e 2e 2e 2e 2e.

$[DLRU, f_* d_*^2]$ (12 moves): (FU UB LF)+(FR BL)(FD BD)(BR)+(LU)+(LD DR RU)+ 6e 2e 2e 2e 2e 6e.

$[UD'B'D^2L', f_* u_* r_*^2]$ (16 moves): (FU DL RF)+(FL FD DR LU BR)+(BU)+(BL)+(BD)+(RU)+ 6e 10e 2e 2e 2e 2e.

$[BU'RD'F'D, b_* r_* d_*]$ (18 moves): (FU)+(FL)+(FR)+(FD DL LB DB)+(BU RU BR LU)+(RD)+ 2e 2e 2e 8e 8e 2e.

$[L'FU'BR, r_* u_* b_*]$ (16 moves): (FU)+(FL LB UB)(FR)+(FD RD DB)+(BR UR UL)(LD)+ 2e 3e 2e 6e 3e 2e.

$[L'F^2D'B, f_* r_*^2 u_*^2]$ (14 moves): (FU BR UR)(FL UL BU RD)(FR)+(FD LD)+(BL)+(BD)+ 3e 4e 2e 4e 2e 2e.

$[U^2D'BR, d_* f_*^2 l_*]$ (14 moves): (FU BL FR)(FL DF)+(BU UL)(BR BD DR)+(LD)+(RU)+ 3e 4e 2e 6e 2e 2e.

23

$[RDUB, l_*u_*^2]$ (12 moves): (FU UB BD DF)(FL)+(FR LD)+(BR)+(BL RU)+(LU DR) 4e 2e 4e 2e 4e 2e.

$[D^2R'L^2, r_*^2d_*f_*^2]$ (12 moves): (FU BU)(FL BR DR)+(FR UL)+(FD BD)(BL LD)+(RU)+ 2e 6e 4e 2e 4e 2e.

$[LDFUF, r_*b_*^2d_*^2]$ (16 moves): (FU BR BL LU BD)+(FL RF)+(FD DR)+(BU)+(LD)+(RU)+ 10e 4e 4e 2e 2e 2e.

$[D'R'BD'F^2, b_*r_*^2u_*]$ (16 moves): (FU)+(FL DR)+(FR UB UL DL)+(FD BR UR)+(BL)+(BD)+ 2e 4e 8e 6e 2e 2e.

$[FU'B^2LD', f_*^2u_*l_*]$ (16 moves): (FU DB RD)+(FL RB LD)+(FR)+(FD)+(BU UR LB)+(LU)+ 6e 6e 2e 2e 6e 2e.

$[B^2UF'D, f_*^2u_*^2l_*]$ (14 moves): (FU BR)+(FL UB FD)(FR UR)+(BL LD UL)(BD)+(RD)+ 4e 3e 4e 3e 2e 2e.

$[FR'DB^2R, u_*^2f_*^2r_*]$ (16 moves): (FU)+(FL DF)+(FR LU)+(BU UR)(BR DB DL)(BL DR)+ 2e 4e 4e 2e 3e 4e.

$[RBU'D, b_*r_*^2d_*]$ (14 moves): (FU DR)+(FL DL)+(FR LU)(FD DB)(BU BL)+(BR RU)+ 4e 4e 2e 2e 4e 4e.

$[L^2D'FR'B^2, f_*u_*l_*]$ (16 moves): (FU)+(FL UL)+(FR BD)+(FD RD)+(BU LD LB RB)+(RU)+ 2e 4e 4e 4e 8e 2e.

$[R'U'L'U^2B', d_*f_*^2r_*]$ (16 moves): (FU FL RB)+(FR FD)+(BU UL)+(BL)+(BD RU RD)+(LD)+ 6e 4e 4e 2e 6e 2e.

$[L^2U^2D'B'R'B^2, r_*u_*b_*]$ (18 moves): (FU DR)+(FL LD)+(FR FD RB)+(BU BL)+(BD UR)+(LU)+ 4e 4e 6e 4e 4e 2e.

[NOT YET]: 4e 4e 4e 4e 4e 4e.

12 edge pieces, 8 cycles

$[D^2RL, b_*r_*^2u_*^2]$ (12 moves): (FU BU)(FL)+(FR)+(FD BD)(BR LB)(LU DR)(LD)+(RU)+ 2e 2e 2e 2e 2e 2e 2e 2e.

$[D'F'R'B', u_*^2l_*f_*^2]$ (14 moves): (FU DB RF BR)(FL DF)(BU)+(BL)+(LU)+(LD)+(RU)+(RD)+ 4e 2e 2e 2e 2e 2e 2e 2e.

$[D^2FU'DR, u_*^2r_*^2b_*]$ (16 moves): (FU FD FL)+(FR)+(BU)+(BR DB)(BL)+(LU)+(LD UR)(RD)+ 6e 2e 2e 2e 2e 2e 2e 2e.

$[RUFBUF', f_*^2r_*u_*^2]$ (18 moves): (FU RF FD BL BD)+(FL)+(BU)+(BR)+(LU)+(LD)+(RU)+(RD)+ 10e 2e 2e 2e 2e 2e 2e 2e.

$[DF'U^2LB'U, u_*f_*r_*]$ (18 moves): (FU)+(FL LD UL)(FR RD BR)(FD)+(BU)+(BL)+(BD)+(RU)+ 2e 3e 3e 2e 2e 2e 2e 2e.

$[BLF'D'BL^2, u_*^2r_*b_*]$ (18 moves): (FU DB LU)(FL)+(FR BU)(FD)+(BR)+(BL RD)+(LD)+(RU)+ 3e 2e 2e 2e 2e 4e 2e 2e.

$[UDFB^2, f_*^2r_*d_*]$ (14 moves): (FU DB)(FL UL)+(FR BL)(FD)+(BU)+(BR UR)+(LD)+(RD)+ 2e 4e 2e 2e 2e 4e 2e 2e.

$[F'L'B'U'B', f_*^2l_*u_*^2]$ (16 moves): (FU BR)+(FL BD UB DF)+(FR)+(BL)+(LU)+(LD)+(RU)+(RD)+ 4e 8e 2e 2e 2e 2e 2e 2e.

$[U^2DR'B'U', f_*^2r_*u_*^2]$ (16 moves): (FU BR BD)+(FL DF UR)+(FR)+(BU)+(BL)+(LU)+(LD)+(RD)+ 6e 6e 2e 2e 2e 2e 2e 2e.

$[L'F'B'L'D', r_*^2d_*f_*^2]$ (16 moves): (FU FR)+(FL BR DB)+(FD)+(BU BL)+(LU)+(LD)+(RU)+(RD)+ 4e 6e 2e 4e 2e 2e 2e 2e.

$[FLU, u_*^2f_*^2l_*^2]$ (12 moves): (FU UR)+(FL LU)+(FR)+(FD)+(BU)+(BR DR)+(BL)+(BD LD)+ 4e 4e 2e 2e 2e 4e 2e 4e.

12 edge pieces, 10 cycles

$[D'R'L'F', u_*^2r_*^2b_*^2]$ (14 moves): (FU)+(FL BL)(FR BR)(FD)+(BU)+(BD)+(LU)+(LD)+(RU)+(RD)+ 2e 2e 2e 2e 2e 2e 2e 2e 2e 2e.

$[L'F^2B'L'F^2U'F', d_*r_*^2f_*^2]$ (20 moves): (FU)+(FL)+(FR BU BL)+(FD)+(BR)+(BD)+(LU)+(LD)+(RU)+(RD)+ 2e 2e 6e 2e 2e 2e 2e 2e 2e 2e.

$[B'RUF'RB'U'R'U^2L', b_*r_*d_*]$ (26 moves): (FU RB)+(FL DB)+(FR)+(FD)+(BU)+(BL)+(LU)+(LD)+(RU)+(RD)+ 4e 4e 2e 2e 2e 2e 2e 2e 2e.

12 edge pieces, 12 cycles

$[R'D'LF'UR'U'F'BU'DBLF'UR'BLD'UB'FD'U, u_*^2r_*^2b_*]$ (54 moves): (FU)+(FL)+(FR)+(FD)+(BU)+(BR)+(BL)+(BD)+(LU)+(LD)+(RU)+(RD)+ 2e 2e 2e 2e 2e 2e 2e 2e 2e 2e 2e 2e.

References

[1] David Singmaster, *Notes on Rubik's 'Magic Cube'*, 5th edition, London, 1980.

1.07C February 22, 2025