

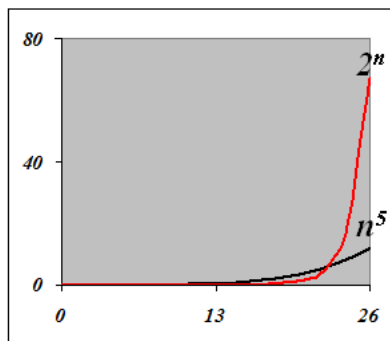
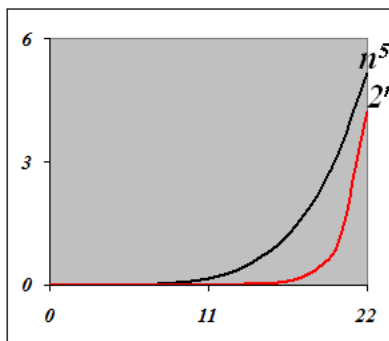
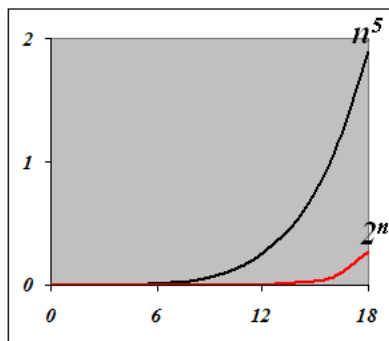


# THEOREM OF THE DAY

## Cook's Theorem on NP-completeness *SATISFIABILITY is NP-complete.*

**SATISFIABILITY:** a group of people are organising a party; is there a guest list which satisfies at least one of each person's stated preferences?

This is an example of a **decision problem**: a simplified theoretical version which just requires a Yes-No answer.



### TIMING CONSIDERATIONS

**Horizontal axis:**  $n$ =number of preferences about party guests;  
**Vertical axis:** time, measured in, say, seconds, required to decide if there is a **satisfying** guest list.

Compare the polynomial  $n^5$ -time solution to the exponential-time  $2^n$  one, as  $n$  increases beyond 20. Exponential-time is always infinitely slower than polynomial-time as problem size becomes infinitely large.

### NONDETERMINISTIC POLYNOMIAL-TIME (NP)

To say a decision problem is in NP means, roughly, that proof for a Yes answer can be written down and checked in polynomial time. Thus, a candidate guest list, e.g. the one on the right, is easily checked as proof of existence of a satisfying guest list.

Guest List
Ben
Costas
Debra
Elroy



The only known solutions to SATISFIABILITY essentially involve checking every possible guest list and this takes exponential time:  $2^n$  steps. The problem is generally believed to lie outside the class P of decision problems with a quick (Polynomial) solution. This remains true even if everybody has exactly three preferences (so-called 3-Satisfiability). But the problem belongs to NP and this property, of admitting polynomially checkable candidates, turns out to be fundamental (unlike the distinction between decision problems and optimisation problems — find an actual guest list! — which is a theoretical convenience).

The theorem appears to refer only to a very specific problem but relates to one of the most famous open questions in mathematics, referred to as P=NP? Cook's 1971 theorem says that if we can find a polynomial algorithm for SATISFIABILITY then the classes P and NP are identical, and we can solve all NP problems in polynomial time, no matter how different from SATISFIABILITY they might appear to be. Leonid Levin published essentially the same result in 1973 and this theorem is also known as the Cook-Levin Theorem.

**Web link:** [rjlipton.wordpress.com/about/](http://rjlipton.wordpress.com/about/): if P=NP? is settled you will read about it here first ... unless you are the one winning the \$1 million: [en.wikipedia.org/wiki/Millennium\\_Prize\\_Problems!](http://en.wikipedia.org/wiki/Millennium_Prize_Problems!)

**Further reading:** *Computers and Intractability* by M.R. Garey and D.S. Johnson, W.H. Freeman & Co Ltd, 1979.

