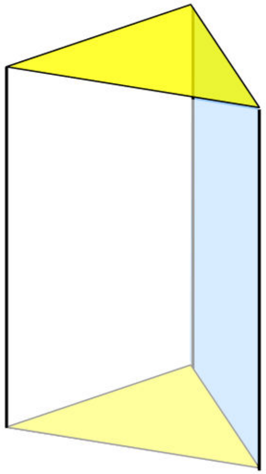
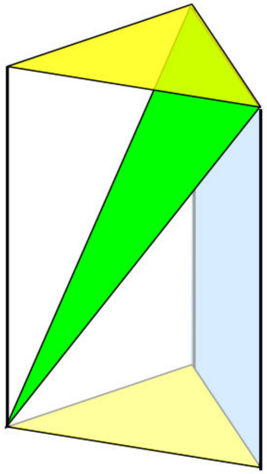
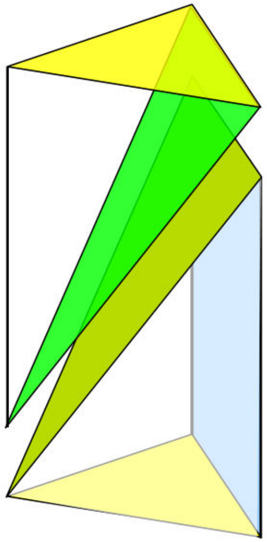
THEOREM OF THE DAY

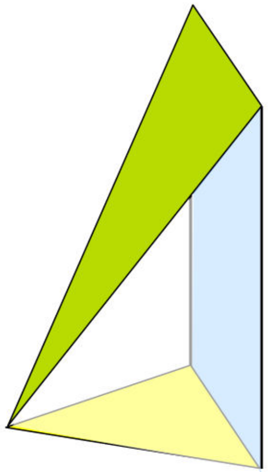
Euclid's Triangular Prism *Any prism with a triangular base is divisible into three triangular-based pyramids of equal volume.*

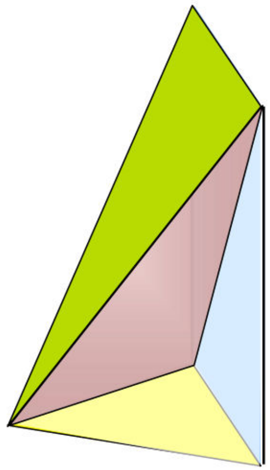


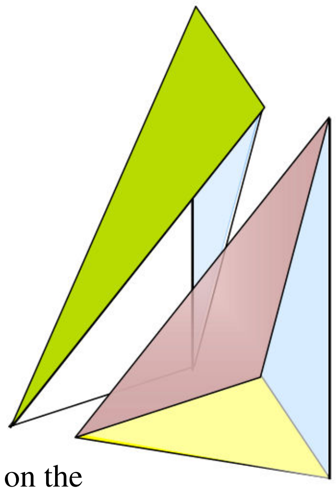




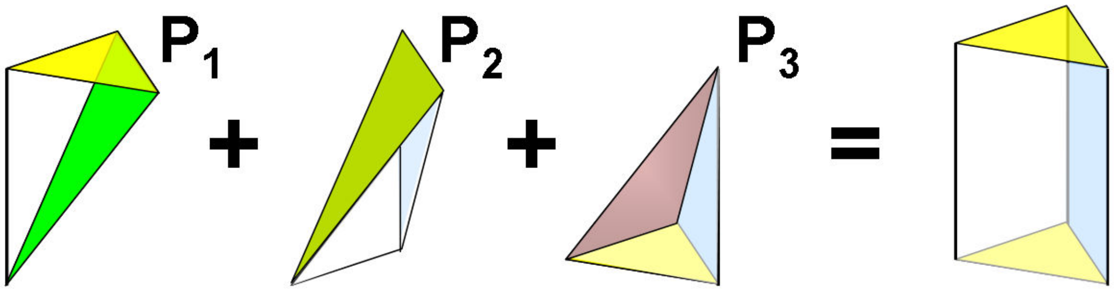
A constructive proof of this proposition is presented in the illustration. The first pyramid (P_1) to be split off here is easily seen to have equal volume to the pyramid (P_3) finally remaining: they share a congruent triangular base and height; and what they exclude (P_2) has the same volume as P_3 since they divide the intact rectangular face of the prism. It must thus also equal P_1 and we conclude, as did Euclid, *Omicron Epsilon Delta*: Oper Edei Deixai (or, in modern usage, "so we are done," or just '□').







This result appears as Proposition 7 in the penultimate book of Euclid's *Elements*, which builds on the foundations of three dimensional geometry laid in Book XI to deal with relative volumes of cones, pyramids, cylinders etc, before Book XIII finally reaches a glorious conclusion with the Platonic Solids. In fact, Euclid adds "*ὅπερ ἔδει δεῖξαι*" only after giving a Corollary: the volume of a triangular pyramid is one third of that of the prism having the same base and height (this remains true for pyramids with arbitrary polygonal bases as proved by Eudoxus of Cnidus in the 4th century BC).



Web link: farside.ph.utexas.edu/books/books.html (links to a dual-language complete *Elements*, nearly 5MB but a truly definitive web resource). Read more on the '□' symbol at www.numericana.com/answer/symbol.htm#halmos.

Further reading: *Euclid's Elements of Geometry* by Richard Fitzpatrick, publ. Richard Fitzpatrick, 2007.