

Birth of a Theorem by Cédric Villani, 2015, Bodley Head, 272 pp, ISBN: 978-1847922526, £18.99.

"The function of a mathematician is ... to add to mathematics, and not to talk about what he or other mathematicians have done." After seventy-five years G.H. Hardy's "melancholy experience" still resonates with, and presumably influences, the profession. Against this we have the fourth of Gian-Carlo Rota's "Ten Lessons I wish I had been Taught" (his well-known 1996 essay): "You are more likely to be remembered by your expository work". Very recently, three of those adding to mathematics at the highest level have bid for Rota's version of posterity. Edward Frenkel's *Love and Math* has just been published in French translation, Michael Harris's *Mathematics Without Apologies* is sure to follow suit; and Cédric Villani's *Théorème Vivant* has gone the other way and we have it in English.

Of the three Villani has the highest claim to what Harris calls 'charisma', having won a Fields medal in 2010. The citation is "for his proofs of nonlinear Landau damping and convergence to equilibrium for the Boltzmann equation." It is the former result which is the hero of his book. The medal ceremony at the International Congress of Mathematicians in Hyderabad is the climax and although there cannot be many readers unaware that Villani's story has a happy ending it is nevertheless a compelling trajectory. An essentially chronological diary format is used and this works well: we share Villani's frustration when childcare or needy colleagues interrupt his work, and his jitters over a seminar he feels he is not ready for; we worry with him that he will miss the strict age limit for a Fields medal; most of all we sympathise as yet another hole appears in yet another version of the enormous proof that is supposed to win the medal. Meanwhile, Villani's enthusiasm is infectious: he loves being at the Institute of Advanced Studies, he loves mixing with already famous scientists, he loves being back in France again, he even embraces the surely crushing commitment of running the Institut Henri Poincaré. Does this make Villani appear pompous or complacent, an obvious risk? Not a bit: certainly he is pleased with himself, but then, he is pleased with everything.

There are frequent short digressions to describe mathematical people and ideas which are related to the story (some less closely than others but a particularly timely aside describes the working relationship between the 2015 Abel prizewinners Nash and Nirenberg, both pioneers in Villani's field). In a less important book these digressions would have been expanded to make up the expected page count for a standard popular mathematics book. But this is not what Villani does.

Instead, and what has attracted most comment in Villani's book, there is the verbatim reproduction of dozens of pages of mathematical text, or in many cases TEX, which is patently unreadable for essentially all of his audience. And 'all' means 'all': interviewed in *Nouvelle Observateur* in September 2012, Villani said (our translation) "Rest assured, most mathematicians will skip these pages too, because they are only intelligible to the tiny minority who work in the exact same subject area as me."

This feature of the book certainly deserves comment. First and foremost the commentary should be Villani's, and he deserves to be quoted at length. Interviewed in October 2012 in *Les Echos* (equivalent to the UK's *FT*) he said of the mathematics (again our translation):

"It was an unusual editorial decision, adopted after careful consideration.

"At no point do I require of my readers to understand these formulae; they

are there simply to act as testimony, like the rough sketches that litter an artist's studio.

"The goal of the book is to reveal to the reader a community, that of mathematicians, in all its sociological aspects: how they work, how and to whom they speak, through what passages of elation and dejection they pass, etc. I wanted above all to reveal their interactions with their colleagues, family, technology, that is everything involved in the forming of an idea, the success of the theorem. It must not be imagined that behind a good mathematical idea there is just a single mathematician who has solved a problem with their single brain: it is a complete ecosystem of human interactions which makes the result possible. If I do not explain the formulae in the book it is precisely because I want the reader *not* to try and understand but rather to focus all their attention on these sociological and human aspects."

Not least, the interaction depicted is that between Villani and Clément Mohout, his former student and coauthor in the proof of nonlinear Landau damping. It is hard to imagine credit sharing achieved with more integrity than is achieved by Villani's verbatim quoting of emails, TEX fragments and typeset mathematics.

A secondary commentary on Villani's structuring of his book must acknowledge his background in French scientific literature and French literature generally. His book bears witness (sometimes to the point of page-skipping at a level to match his formulae!) of an extensive and eclectic cultural background. He is avowedly indebted to Poincaré as a scientist but also as a communicator (Poincaré on science continues to be mainstream reading in France, with prefaces by mainstream modern French philosophers). Villani is following in the literary footsteps of Alain Connes: his *Mathématiques en liberté* (2012), coauthored with three other mathematicians and a philosopher of science, copying the template of Connes' *Matière à Pensée* (1989). He has been compared with the French poet and mathematician Jacques Roubaud (*Mathématique*, 1997) who belongs to the elite experimental writing group Oulipo. And now he is breaking into another francophone subculture by scripting *Les Rêveurs lunaires*, a comic art book on World War II mathematicians (including, naturally, Alan Turing: "C'est là à Bletchley Park, que j'ai gagné mon surnom de 'Prof'"). In this context, Villani's presentation of mathematical life, rather than being dismissed as eccentric or self-indulgent, is to be judged as conceptual and innovative.

Even in France it is hard to imagine *Birth of a Theorem* remaining in print as long as has Poincaré. In themselves, his 'raw' mathematics and even his 'complete ecosystem' of mathematical interactions will inevitably come to seem very dated, although a gift to future historians of mathematics. But Villani's is a brave and ground-breaking book: it answers the non-scientist's "What do mathematicians do?" more honestly than anyone ever has. Yes, there are those dense pages where the answer is brutally honest! But Villani compensates with humanity and a strong narrative and a *joie de vivre* that will disarm all but the most resentful. We must pray that his commitments allow him to continue to pacify the spirit of G.H. Hardy and that we can look forward to *Birth of a Corollary*.