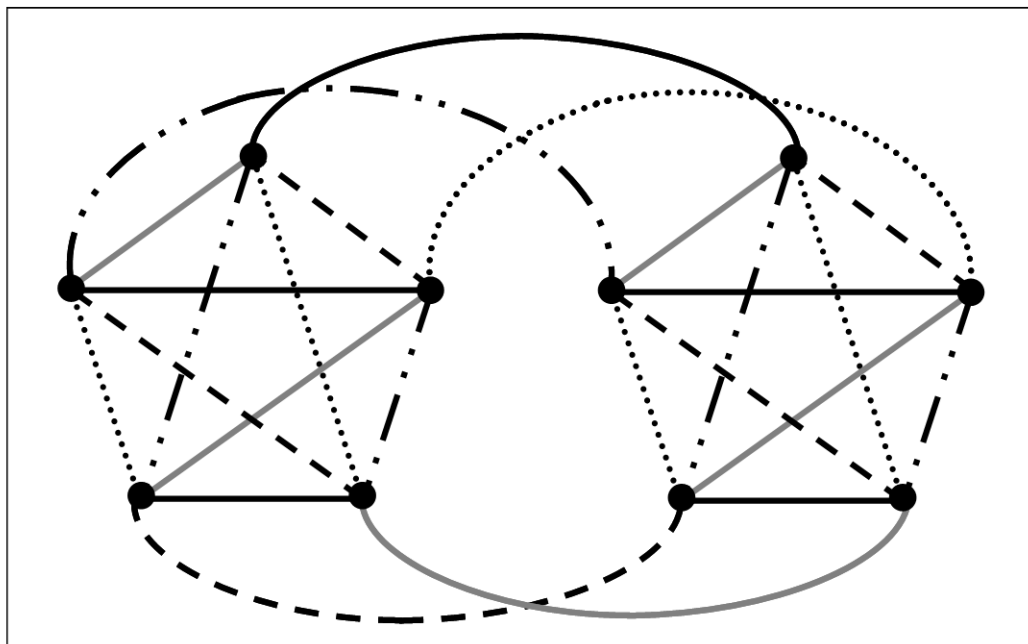


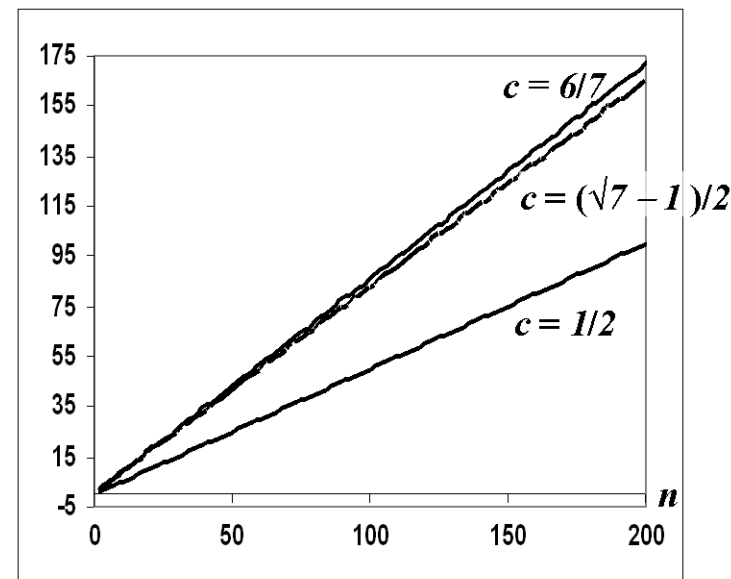


# THEOREM OF THE DAY

**1-Factorisation of Regular Graphs (a Theorem Under Construction!)** *There exists a constant,  $c$ , such that all simple  $d$ -regular graphs of even order,  $n$ , with  $cn \leq d$ , have a 1-factorisation.*



**A 1-factorisation of  $K_2 \times K_5$  ( $n = 10, d = n/2$ )**



**Comparison of  $cn$  with increasing  $n$**

The graph  $K_2 \times K_t$  (that is, two copies of  $K_t$  with edges joining corresponding vertices) has a 1-factorisation for all  $t$  with  $1 \leq t$ , and is  $d$ -regular with  $d = cn$ ,  $c = 1/2$  (that is, every vertex is incident with  $n/2$  edges). This is illustrated, above left, for  $t = 5$  (with, what is more, a *perfect* 1-factorisation: any pair of edge-colours produces a Hamiltonian circuit in the graph); but the ultimate goal of  $c = 1/2$  is well below what has been achieved, so far, for general  $d$ -regular graphs, as the right-hand chart shows.

## Construction notes:



- 1985: Amanda Chetwynd and Anthony Hilton prove existence of  $c$  by showing  $c \leq 6/7$ . They conjecture that  $c = 1/2$  is best possible.
- 1985: R. Häggkvist proves  $\forall \epsilon$ , can take  $c = 1/2 + \epsilon$  for large enough  $n$  ('97: published independently, Perković & Reed).
- 1989: Chetwynd and Hilton achieve  $c = (\sqrt{7}-1)/2 \approx 24/29$  (as do Niessen and Volkmann independently, 1990).
- 2004: Hilton's student David Cariolaro achieves  $c = (\sqrt{57}-3)/6 \approx 22/29$ , except for 2 special classes of  $d$ -regular graphs.
- 2013: Béla Csaba, Daniela Kühn, Allan Lo, Deryk Osthus and Andrew Treglown prove the conjecture for large  $n$  (removing the  $\epsilon$  from Häggkvist and Perković & Reed's 1985 result).

**Web link:** Allan Lo's talk (1MB pdf) from [www.maths.dur.ac.uk/events/Meetings/LMS/2013/GTI13/talks.html](http://www.maths.dur.ac.uk/events/Meetings/LMS/2013/GTI13/talks.html)

**Further reading:** *Graph Edge Coloring: Vizing's Theorem and Goldberg's Conjecture* by Michael Stiebitz, Diego Scheide, Bjarne Toft and Lene M. Favrholdt, Wiley-Blackwell, 2012, Chapter 9.

