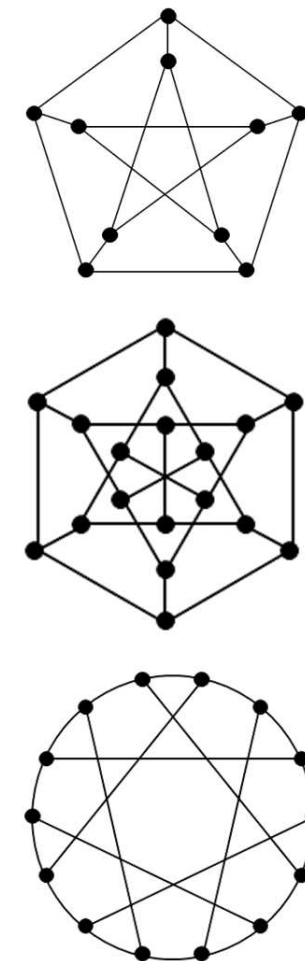
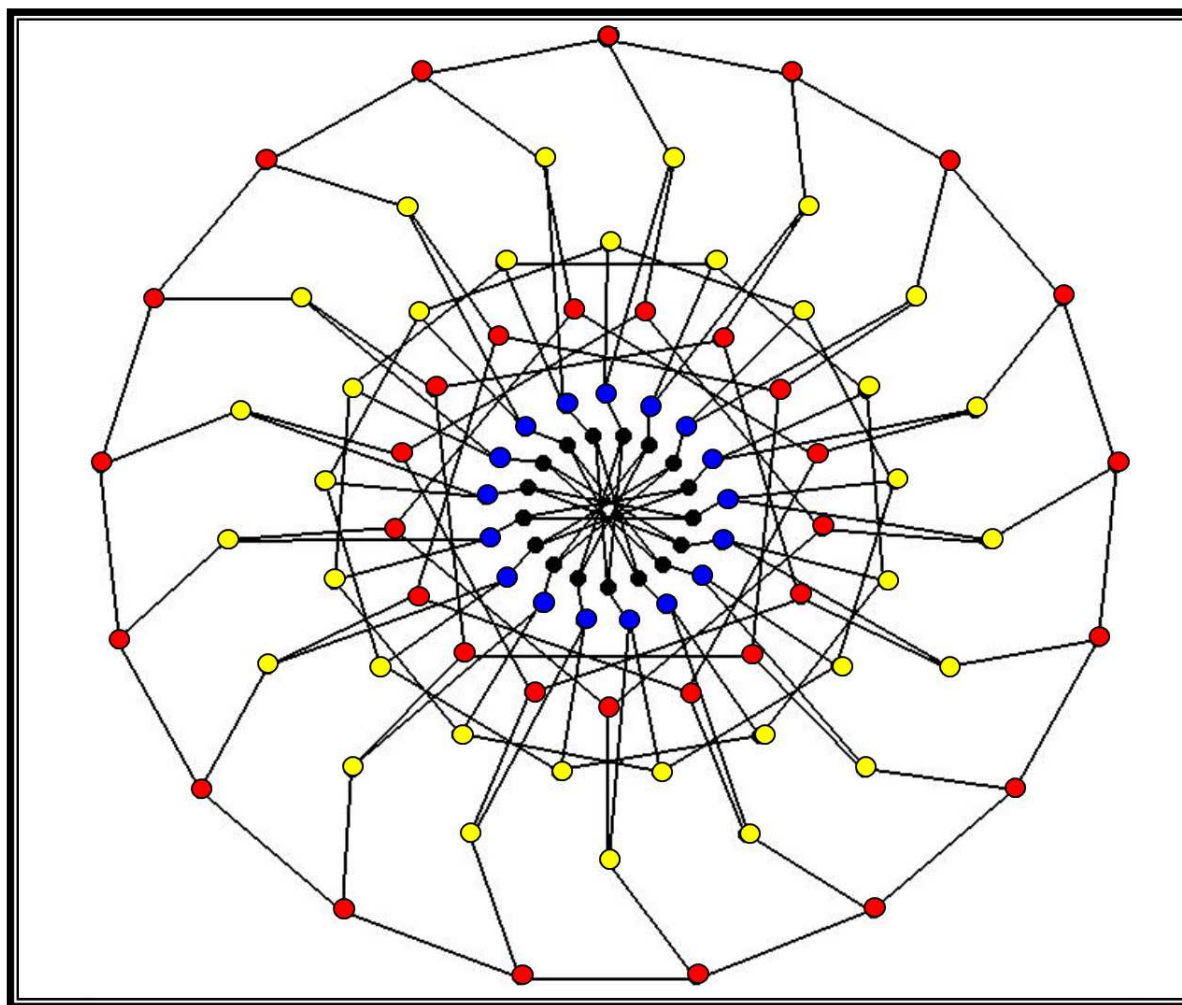
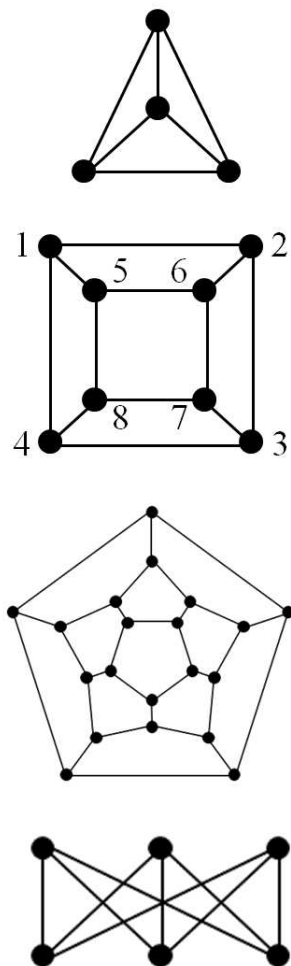




# THEOREM OF THE DAY

**Cameron's Theorem on Distance Transitive Graphs** *For any integer  $k > 2$  there exist only finitely many finite distance-transitive  $k$ -regular graphs.*



Distance transitivity means: if vertices  $a$  and  $b$  are the same (minimum) distance apart as  $\alpha$  and  $\beta$  then there is a permutation of all the vertices which takes  $a$  to  $\alpha$  and  $b$  to  $\beta$  and which preserves all edge relationships. For example, the graph of the cube is shown, labelled, above left. Vertices 1 and 7 are at distance 3 and so are vertices 4 and 6. And indeed the permutation  $(1\ 4)(2\ 3)(5\ 8)(6\ 7)$  preserves edge relationships while interchanging the positions of the vertex pairs (1, 7) and (4, 6). A number of distance-transitive 3-regular graphs (every vertex adjacent to precisely 3 others) are shown above. The Biggs–Smith graph in the centre is the largest. It has 102 vertices which have been colour-coded here according to their distance from the innermost circle of 17 (black) vertices.

Norman Biggs and Derek H. Smith proved in 1971 that there are exactly twelve 3-regular distance transitive graphs. It is at first sight very surprising that even a strong condition on symmetry should defeat the variety available in arbitrarily large graphs. Peter Cameron's deep theorem (1982) shows that this defeat applies even when arbitrarily many adjacencies are allowed.

**Web link:** [www.math.carleton.ca/~robertb/dtg.pdf](http://www.math.carleton.ca/~robertb/dtg.pdf) (1.9MB) (see chapter 10, I adapted the drawing of the Biggs-Smith graph from p. 116).  
**Further reading:** *Algebraic Graph Theory (2nd Edition)* by Norman Biggs, C.U.P., 1994.

