



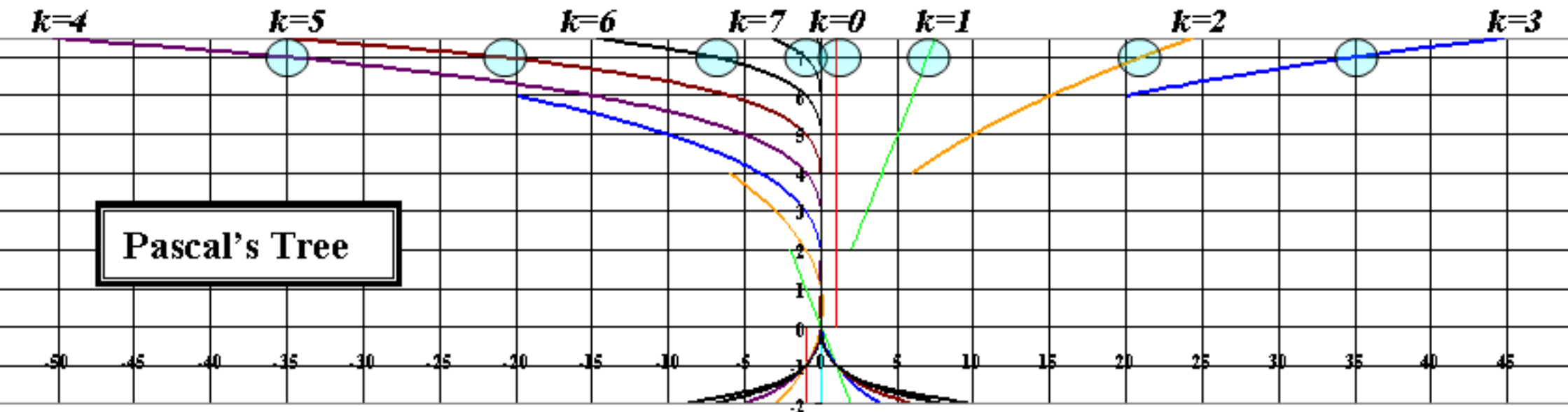
THEOREM OF THE DAY



The Binomial Theorem For n a positive integer and real-valued variables x and y ,



$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k.$$



Given n distinct objects, the binomial coefficient $\binom{n}{k} = n!/k!(n-k)!$ counts the number of ways of choosing k . Transcending its combinatorial role, we may instead write the binomial coefficient as: $\binom{n}{k} = \frac{n}{k} \times \frac{n-1}{k-1} \times \dots \times \frac{n-(k-1)}{1}$; taking $\binom{n}{0} = 1$. This form is defined when n is a real or even a complex number. In the above graph, n is a real number, and increases continuously on the vertical axis from -2 to 7.5 . For different values of k , the value of $\binom{n}{k}$ has been plotted but with its sign reversed on reaching $n = 2k$, giving a discontinuity. This has the effect of spreading the binomial coefficients out on either side of the vertical axis: we recover, for integer n , a sort of (upside down) Pascal's Triangle. The values of the triangle for $n = 7$ have been circled.

If the right-hand summation in the theorem is extended to $k = \infty$, the result still holds, provided the summation converges. This is guaranteed when n is an integer or when $|y/x| < 1$, so that, for instance, summing for $(4 + 1)^{1/2}$ gives a method of calculating $\sqrt{5}$.

The binomial theorem may have been known, as a calculation of poetic metre, to the Hindu scholar Pingala in the 5th century BC. It can certainly be dated to the 10th century AD. The extension to complex exponent n , using generalised binomial coefficients, is usually credited to Isaac Newton.

Web link: arxiv.org/abs/1105.3513

Further reading: *A Primer of Real Analytic Functions, 2nd ed.* by Steven G. Krantz and Harold R. Parks, Birkhäuser Verlag AG, 2002, section 1.5.

