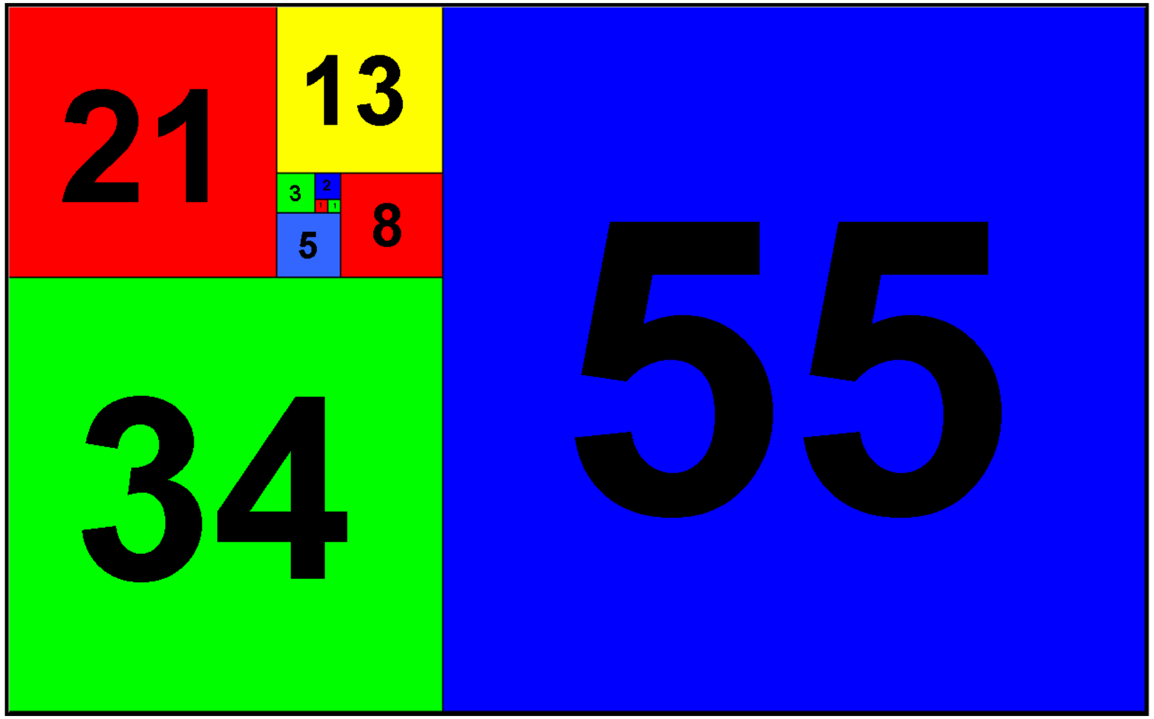
THEOREM OF THE DAY

Binet's Formula *Let* φ *denote the* golden ratio, $\frac{1}{2}\left(1+\sqrt{5}\right)$, *whose first 100 decimal places are:*

$\varphi = 1.6180339887\ 4989484820\ 4586834365\ 6381177203\ 0917980576\ 2862135448\ 6227052604\ 6281890244\ 9707207204\ 1893911374\ldots$

Let $(F_i)_{i\geq 0}$ *be the Fibonacci sequence:* $F_0 = 0$, $F_1 = 1$ *and, for* $k \geq 2$, $F_k = F_{k-1} + F_{k-2}$. *Then, for* $n \geq 0$,

$$F_n = \frac{1}{\sqrt{5}}\left(\varphi^n - (-\varphi)^{-n}\right).$$



The first 10 nonzero terms of the Fibonacci sequence are depicted here as a spiral of adjoining squares each having side length the sum of the previous two. The ratio of each side to its predecessor, F_n/F_{n-1}, tends to φ. Now φ happens to be marginally greater than the ratio of 1 mile to 1 km (≈ 1.609). So if M is a number of miles, an integer, and is written down as a sum of Fibonacci numbers (always possible — the Fibonacci sequence is what is called *complete*) then M is converted to kilometres by shifting each Fibonacci number in the sum up to its successor. E.g. $M = 72$m is written $M = 1 + 3 + 13 + 55$. Shifting up gives $2 + 5 + 21 + 89 = 117$km (the actual figure is nearer to 116).

The formula was published by Jacques Philippe Marie Binet in 1843 but was known, in the 18th century, to Daniel Bernoulli, Leonhard Euler and Abraham de Moivre. But its proudest moment, surely, came in the 20th in 1971, with its role in the resolution of Hilbert's 10th problem.

Web link: www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/

Further reading: *The Golden Ratio and Fibonacci Numbers* by R.A Dunlap, World Scientific, 1998.