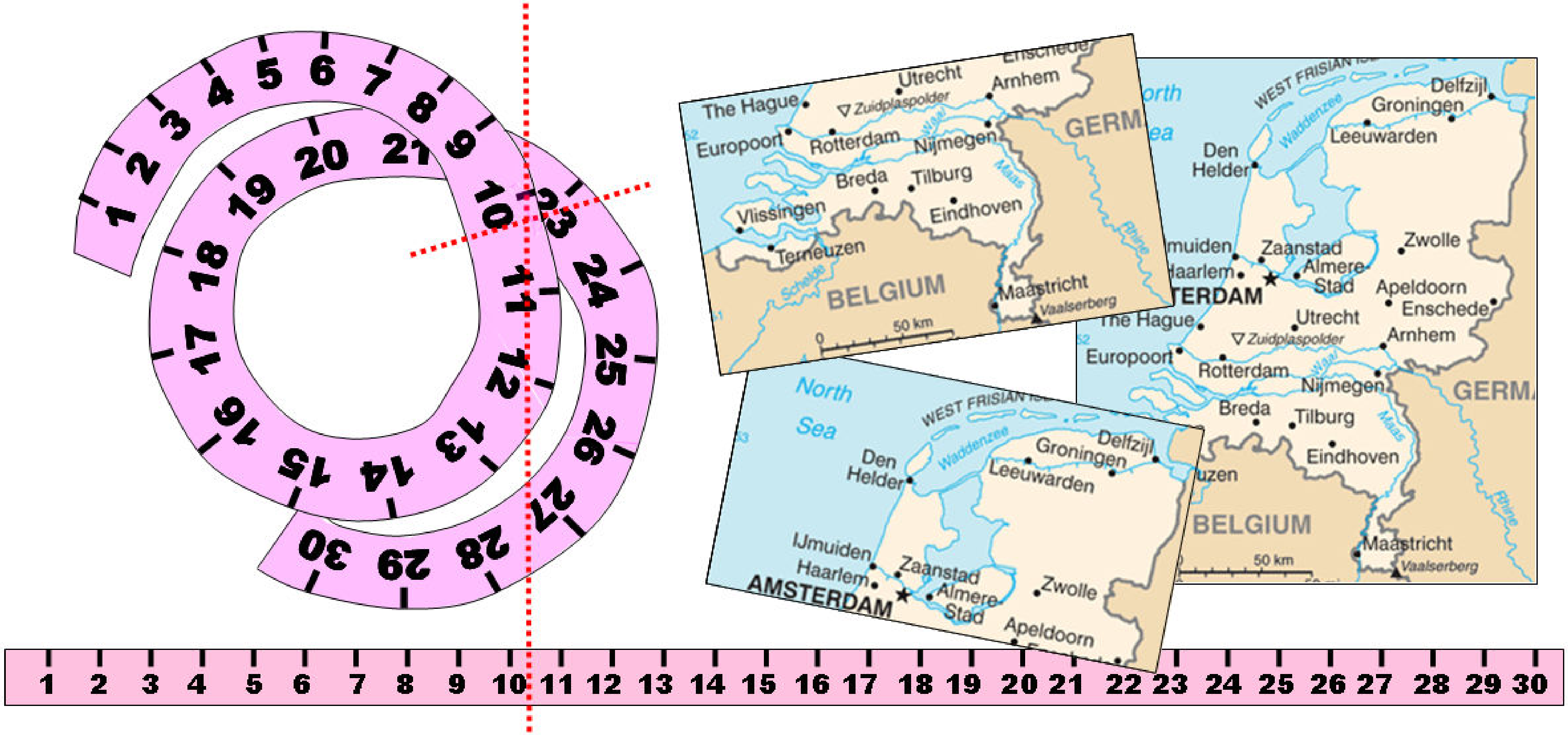
THEOREM OF THE DAY

Brouwer's Fixed Point Theorem *Let B^n denote the n-dimensional closed unit ball and let $f : B^n \longrightarrow B^n$ be a continuous function. Then f has a fixed point: for some $x \in B^n$, $f(x) = x$.*



In one dimension, B^n is just the interval $[0, 1]$ consisting of all real numbers from 0 to 1. Brouwer's theorem can be visualised in terms of two tape measures (say, cloth ones, as used by dressmakers). The function f twists and folds one of the tape measures; which nevertheless retains at least one point aligned with the other measure. In two dimensions B^n is the solid disk of diameter one; this time the illustration consists of two copies of a map with one twisted and folded by f. Some location will again remain common to both maps. But f must be **continuous**: no breaks or jumps. If we cut a map of the Netherlands in half and invert top and bottom halves then no locations are left fixed.

L.E.J. Brouwer was a leading figure in the early 20th century movement to make mathematics *constructive*: to avoid, for example, proofs by contradiction which eliminate the possibility of something being false without explaining why it must be true. There are many important non-constructive proofs of existence and Brouwer's 1910 Fixed Point Theorem is, ironically, one of them: it tells you there is a fixed point but gives you no way of finding it.

Web link: rjlipton.wordpress.com/2009/04/ (the first entry). Map from www.lib.utexas.edu/maps/.

Further reading: *Five Golden Rules: Great Theories of 20th Century Mathematics and Why They Matter* by John L Casti, Wiley, 1997.