

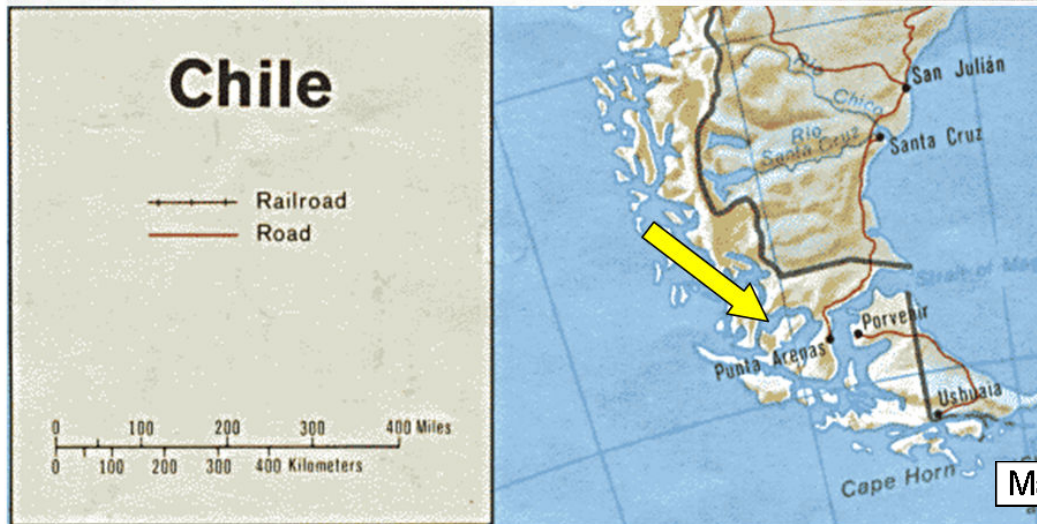


# THEOREM OF THE DAY

**The Borsuk-Ulam Theorem** *Let  $f : S^n \rightarrow \mathbb{R}^n$  be a continuous map. Then some pair of antipodal points on  $S^n$  is mapped by  $f$  to the same point in  $\mathbb{R}^n$ .*

For  $n = 2$ , this theorem can be interpreted as asserting that some point on the globe has precisely the same weather as its antipodal point. The ‘weather’ has to mean two variables ( $\mathbb{R}^2$ ) that vary continuously ( $f$ ) on the surface ( $S^2$ ) of the earth. Perhaps temperature and humidity will do? The maps (right) show a pair of antipodal points with, far right, the nearby August weather as given by the WMO.

As you can see, I have quite failed to match up the weather! Clicking on either map will take you to a nice website where any location is paired with its antipodal point: try if you can do better than me!



Forecasts from <http://worldweather.wmo.int>

## Ulan Ude

### Weather Forecast

Issued at 12:49 (Local time) 28 Aug 2008

Date	Temperature °C		Weather
	Minimum	Maximum	
29 Aug ( Fri )	6	19	FINE DRY
30 Aug ( Sat )	4	19	FINE DRY
31 Aug ( Sun )	6	23	FINE DRY

## Punta Arenas

### Weather Forecast

Issued at 18:00 (Local time) 27 Aug 2008

Date	Temperature °C		Weather
	Minimum	Maximum	
28 Aug ( Thu )	-4	0	SHOWERS
29 Aug ( Fri )	-3	1	SHOWERS
30 Aug ( Sat )	-2	2	SHOWERS

Maps from [www.lib.utexas.edu/maps](http://www.lib.utexas.edu/maps)

“While chatting at the Scottish Café with Borsuk, an outstanding Warsaw topologist, he [Ulam] saw in a flash the truth of what is now called the Borsuk-Ulam theorem. Borsuk had to commandeer all his technical resources to prove it.”

**Web link:** [www.math.hmc.edu/~su/papers.dir/borsuk.pdf](http://www.math.hmc.edu/~su/papers.dir/borsuk.pdf). The Ulam story is from [www.emis.de/newsletter/current/current9.pdf](http://www.emis.de/newsletter/current/current9.pdf).

**Further reading:** *Basic Topology* by M.A. Armstrong, Springer-Verlag New York Inc, 1997, Chapter 9.

