THEOREM OF THE DAY

The Robbins Problem *Every Robbins algebra is a Boolean algebra.*

	BOOLEAN ALGEBRA	
Commutation	$a+b=b+a$	$a\times b=b\times a$
Association	$(a+b)+c=a+(b+c)$	$(a\times b)\times c=a\times(b\times c)$
Zero	$0+a=a+0=a$	$0\times a=a\times 0=0$
One	$1+a=a+1=1$	$1\times a=a\times 1=a$
Distribution	$a+(b\times c)=(a+b)\times(a+c)$	$a\times(b+c)=(a\times b)+(a\times c)$
Absorption	$a\times(a+b)=a+(a\times b)=a$	
Complementation	For every a there is some a' satisfying $a\times a'=0$ and $a+a'=1$	

$\Rightarrow$

?? $\Leftarrow$

	ROBBINS ALGEBRA
Commutation	$a+b=b+a$
Association	$(a+b)+c=a+(b+c)$
Robbins' Axiom	$((a+b)'+(b+c')')'=a$

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----- EQP 0.9, June 1996 -----
The job began on eyas09.mcs.anl.gov, Wed Oct  2 12:25:37 1996
UNIT CONFLICT from 17666 and 2 at 678232.20 seconds.

---------------- PROOF ----------------

2 (wt=7) [] -(n(x + y) = n(x)).
3 (wt=13) [] n(n(n(x) + y) + n(x + y)) = y.
5 (wt=18) [para(3,3)] n(n(n(x + y) + n(x) + y) + y) = n(x + y).
6 (wt=19) [para(3,3)] n(n(n(n(x) + y) + x + y) + y) = n(n(x) + y).
24 (wt=21) [para(6,3)] n(n(n(n(x) + y) + x + y + y) + n(n(x) + y)) = y.
(wt=29) [para(24,3)] n(n(n(n(n(x) + y) + x + y + y) + n(n(x) + y) + z) + n(y + z)) = z.
(wt=27) [para(24,3)] n(n(n(n(x) + y) + n(n(x) + y) + x + y + y) + y) = n(n(x) + y).
(wt=29) [para(48,3)] n(n(n(n(x) + y) + n(n(x) + y) + x + y + y + y) + n(n(x) + y)) = y.
(wt=34) [para(47,3)] n(n(n(n(n(x) + y) + x + y + y) + n(n(x) + y) + n(y + z) + z) + z) = n(y + z).
(wt=42) [para(250,3)] n(n(n(n(n(n(x) + y) + x + y + y) + n(n(x) + y) + n(y + z) + z) + z + u) + n(n(y + z) + u)) = u.
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News Flash!

McCune, Veroff, Fitelson, Harris, Feist and Wos publish short single-axiom specification of Boolean Algebra, *J. Automated Reasoning*, 29(1), 1–16, 2002:

$$(((x+y)'+z)'+(x+(z'+(z+u)')')')'=z$$

A ***Boolean algebra* is a set containing** at least two elements, 0 (zero) and 1 (one); closed under three operations, +, × and ′(complement); and satisfying the laws given in the table, above left. These are exactly the laws of ordinary (integer) arithmetic, until we reach the four that are shaded (red), which take us into a different realm — that of logic and set theory. Amazingly, this whole realm may then be collapsed down to just three laws, specifying the *Robbins algebra*, above top right. Now, while it is simple to see that any Boolean algebra is a Robbins algebra, the reverse derivation is extraordinarily difficult. The inset, bottom right, is part of a summary of the proof, by the theorem prover EQP, that every Robbins algebra satisfies the so-called *2nd Winker condition*: $\exists C, D.(C+D)'=C'$, already known to guarantee the laws of Boolean algebra.

Herbert Ellis Robbins (1915–2001), better known as a statistician, influenced logic, mathematics and computer science by proposing a slight simplification of an algebra due to Edward Vermilye Huntington in 1933. Proving that this was equivalent to Boolean algebra, the *Robbins Problem*, stone-walled human logicians for decades. A breakthrough came in 1979, when Larry Wos and Steve Winker at the Argonne National Laboratory, using the fledgling methods of automated reasoning, discovered the Winker conditions. Even then, it took nearly twenty more years before William McCune, in collaboration with his EQP theorem prover, finally completed the solution in October 1996. This was a major milestone in artificial intelligence.

Web link: www.cs.unm.edu/~mccune/papers/robbins/ (links to the EQP output which is reproduced in part above).

Further reading: *Automated Reasoning and Its Applications: Essays in Honor of Larry Wos* by Robert Veroff (ed.), MIT Press, 1997.