THEOREM OF THE DAY

The Second and Third Isomorphism Theorems *Suppose H is a subgroup of G and K is a normal subgroup of G. Then*

2nd Isomorphism Theorem: *HK is a subgroup of G and $H \cap K$ is a normal subgroup of H, and*

$$HK/K \cong H/(H \cap K).$$

Suppose, that H is also normal in G and that K is contained in H. Then

3rd Isomorphism Theorem: *K is normal in H, and*

$$(G/K)/(H/K) \cong G/H.$$

2nd Isomorphism Theorem **3rd Isomorphism Theorem**

The second and third isomorphism theorems look seductively like the rules for fractions: you can 'multiply' the top and bottom using $\cap$ without changing the value ($\frac{x}{y} = \frac{a \times x}{a \times y}$); and you can cancel ($\frac{x}{y} \times \frac{y}{z} = \frac{x}{z}$). This similarity is best treated as no more than a mnemonic, however!

Normal subgroups, whose cosets themselves form a group under the natural multiplication, are a way of breaking down the structure of large groups into smaller ones. Simple groups, those having no normal subgroups, play a somewhat analogous role to the primes in number theory. Unlike the primes, however, they have been completely catalogued, this so-called *classification of the finite simple groups* being one of the great achievements of twentieth century mathematics. These theorems, like the First Isomorphism Theorem, may be attributed to Emmy Noether who, as a great architect of twentieth century algebra, gave them a secure place in the foundations of the edifice.

Web link: www.math.uic.edu/~radford/math516f06/IsoThms.pdf

Further reading: *Symmetry and the Monster: One of the Greatest Quests of Mathematics* by Mark Ronan, Oxford University Press, 2006.