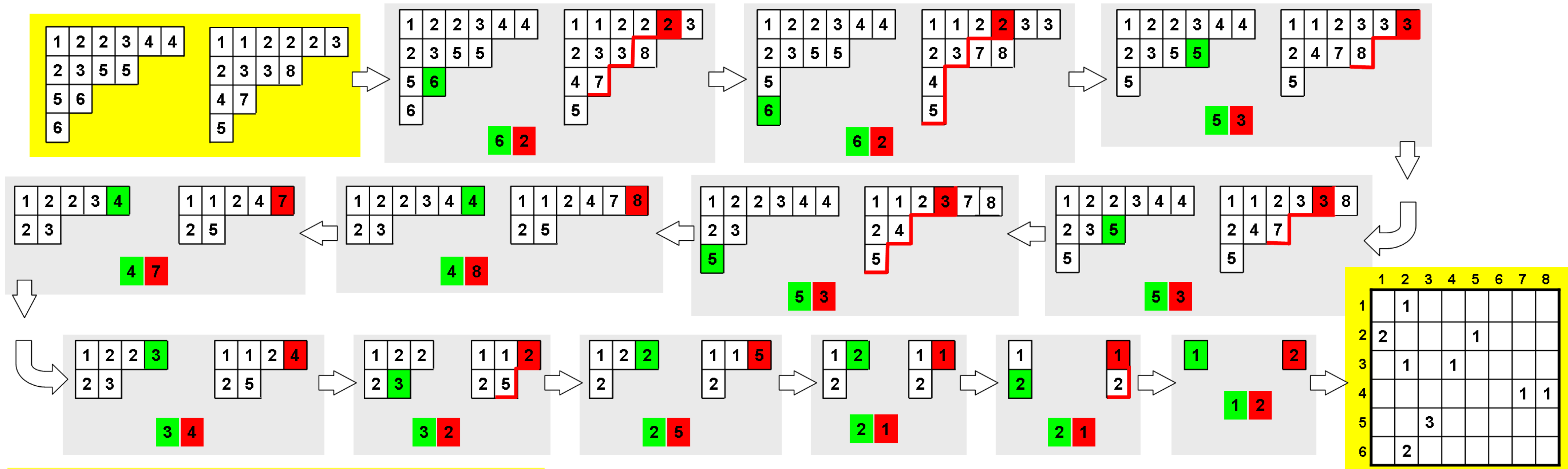
THEOREM OF THE DAY

The Robinson–Schensted–Knuth Correspondence *There is a one-to-one correspondence between ordered pairs of equal-shaped, semistandard Young tableaux comprising N cells, and having largest entries m and n, respectively, and nonnegative $m \times n$ integer matrices whose entries sum to N.*



Alphabetically	each	line	must	steadily
Ascend;	equally	mayst	no	syllable
Atop	inferior	subtend;	superiorly	wide,
Contrariwise,	rows	than	those	
Our				
Poem				
Subsequently				
Supplies.				

A *standard Young tableau* has N cells left-justified in rows of non-increasing length. The cells contain the numbers $1 \ldots N$ with each row and column increasing.

In *semistandard* Young tableaux, entries may be repeated and rows, but not columns, may be non-decreasing, rather than strictly increasing. In the example above, a pair of such tableaux, on 13 cells, is converted to a matrix whose entries sum to 13. In each step of the conversion some matrix entry is increased by one, working from bottom right to top left. At each step the largest value in the left-hand tableau cell determines the matrix row; the rightmost occurrence of this value indicates a ‘starting cell’ in the right-hand tableau. The value in this starting cell then replaces, in the row above, the rightmost value than which it is bigger; and so on through the rows of the tableau, the value replaced in row 1 being the index in the matrix row at which the unit increase will occur. The whole process is reversible, so that matrices, conversely, yield tableaux pairs.

Young tableaux were introduced by Alfred Young in 1900 as tool in group theory. The rule described above was first devised for standard tableau by Gilbert de Beauregard Robinson in 1938. It was rediscovered in 1961 by Craig Schensted and rediscovered again, but in a much more general and complete form, by Donald E. Knuth in 1970.

Web link: mps2016.labri.fr/archives/krattenthaler.pdf

Further reading: *Combinatorics of Permutations* by Miklós Boná, Chapman and Hall, 2004, Chapter 7.