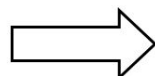
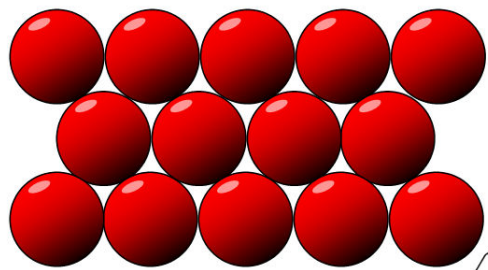




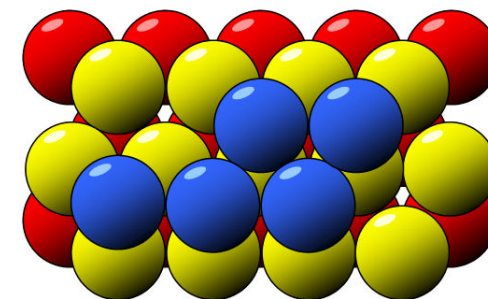
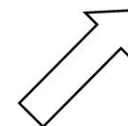
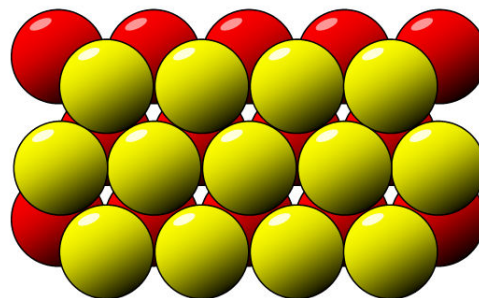
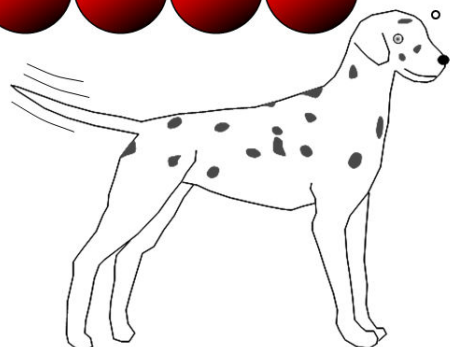
# THEOREM OF THE DAY

**Kepler's Conjecture** Any packing of three-dimensional Euclidean space with equal-radius spheres has density bounded by  $\tau \sqrt{2}/12 \approx 0.74$  ( $\tau = 2\pi$ ).

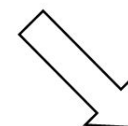
The *face-centred cubic packing* aligns the spheres on a three-dimensional square grid or *lattice*; in *hexagonal close packing* no three layers align sphere centres but there is still regular structure and both packings achieve the density bound of Kepler's conjecture, that is, the total empty space between balls totals  $1 - \tau \sqrt{2}/12$ , or a little less than 26%. But some non-regular arrangements do better for partial packings of three-space; ruling out such irregular packings makes Kepler's conjecture particularly difficult to prove.



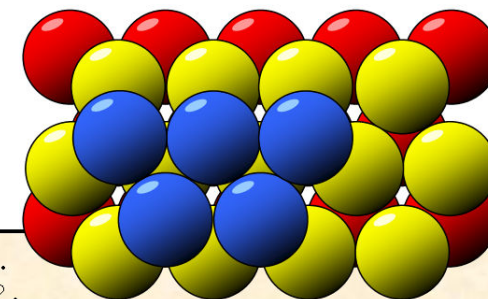
I'm seeing spots!



**Face-centred cubic packing:** layer three (blue) offset vertically against layer one (red)



**Hexagonal close packing:** layer three (blue) vertically eclipses layer one (red)



**Historical notes:** 1611: Johannes Kepler states conjecture in response to question of Thomas Harriot.  
 1890: Axel Thue proves the 2D analogue - optimal circle packing has density  $\sqrt{3}/12$ .  
 1900: Hilbert makes Kepler's Conjecture one of his challenges for the 20<sup>th</sup> century - part of his '18<sup>th</sup> problem'.  
 1953: László Fejes Tóth reduces proof of conjecture to large but well-defined set of calculations.  
 1998: Thomas Hales (with grad. student Samuel P. Ferguson) announces computer-aided solution to these calculations.  
 2003: Hales launches FlySpeck project to verify computer proof using automated theorem-checkers (non-computer part of proof now generally accepted).  
 2012: Hales' *Dense Sphere Packings* provides accessible, self-contained version of his proof, by now unassailable.  
 2014: Flyspeck completes on August 14.



**Web link:** Hales's 2000 overview for the AMS is the best introduction: click on *Cannonballs and Honeycombs* at [www.ams.org/notices/200004](http://www.ams.org/notices/200004). The official report on the Flyspeck project is on the arxiv at [arxiv.org/abs/1501.02155](http://arxiv.org/abs/1501.02155).

**Further reading:** *Kepler's Conjecture* by George Szpiro, John Wiley & Sons, 2003; and *Dense Sphere Packings* by Thomas C. Hales, CUP, 2012.

