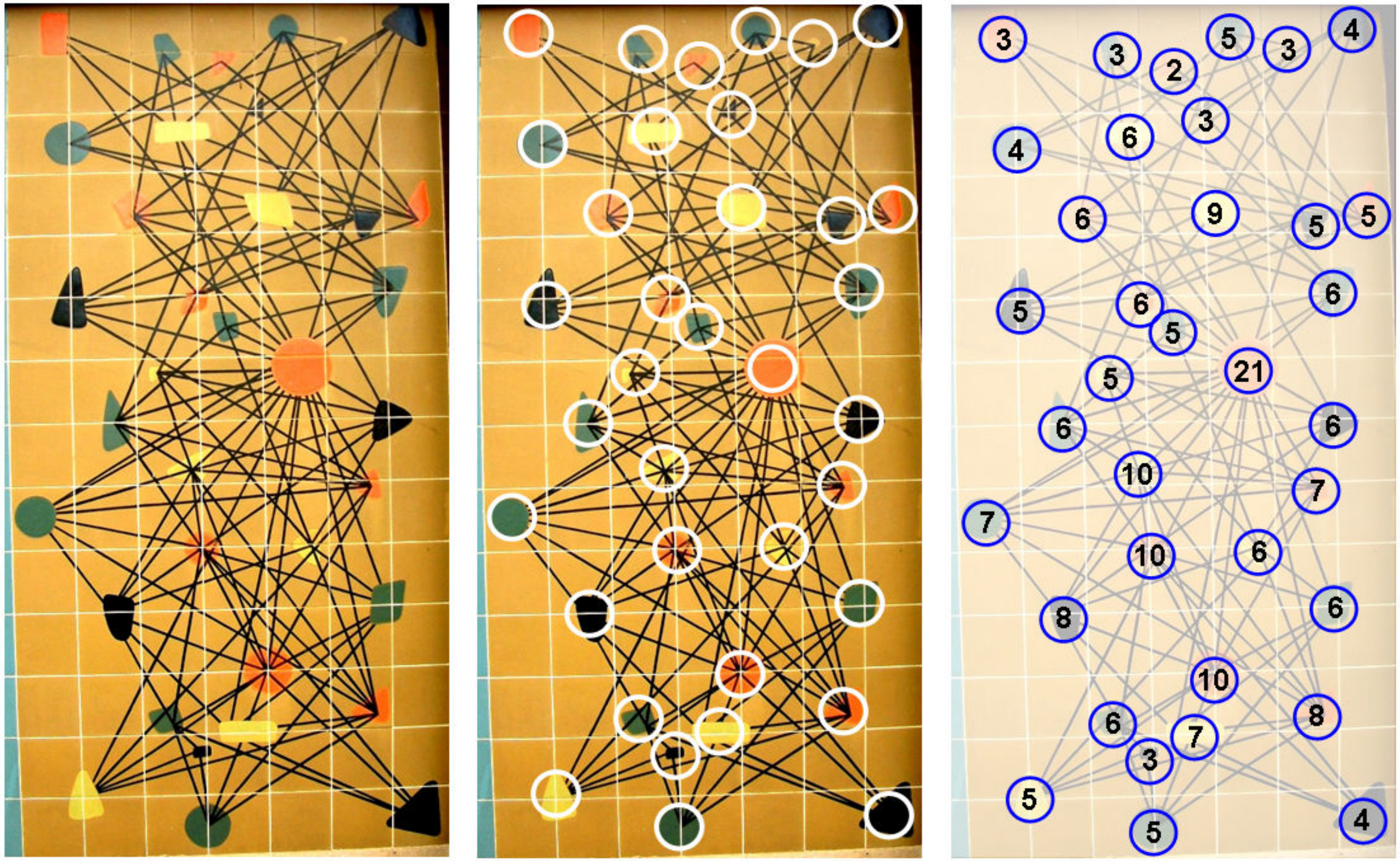
THEOREM OF THE DAY

The Handshaking Lemma *In any graph the sum of the vertex degrees is equal to twice the number of edges.*



The *degree* of a vertex is the number of edges incident with it (a *self-loop* joining a vertex to itself contributes 2 to the degree of that vertex). Suppose that vertices represent people at a party and an edge indicates that the people who are its end vertices shake hands. If everybody counts how many times they have shaken hands then the grand total will be twice the number of handshakes: there are twice as many hands shaken as handshakes. This rather obvious truth is nevertheless a key which unlocks some far from obvious facts. For example, a *3-regular* graph, in which every vertex has degree 3, must have an even number of vertices. Not hard to prove, but a triviality given the immediate corollary of the Handshaking Lemma that the number of odd degree vertices in any graph must be even. Some still more impressive examples are given in the weblinks below. The image above illustrates an application, to count the edges in the graph above left, which is more humble; nevertheless, without distinguishing the vertices (centre) and applying the Handshaking Lemma (right) you would surely go crazy before discovering the answer!

The Handshaking Lemma has its origins in Leonhard Euler's famous 1736 analysis of the 'Bridges of Königsberg' problem.

Web link: mathoverflow.net/questions/77906. The Euler story is here: www.maa.org/programs/maa-awards/writing-awards/the-truth-about-konigsberg.
The tiling which is the basis for our illustration was photographed at Queen Mary University of London (click on the icon, top right, for more details).

Further reading: *Graph Theory: 1736–1936* by Norman L. Biggs, E. Keith Lloyd and Robin. J. Wilson, Clarendon Press, 1986.