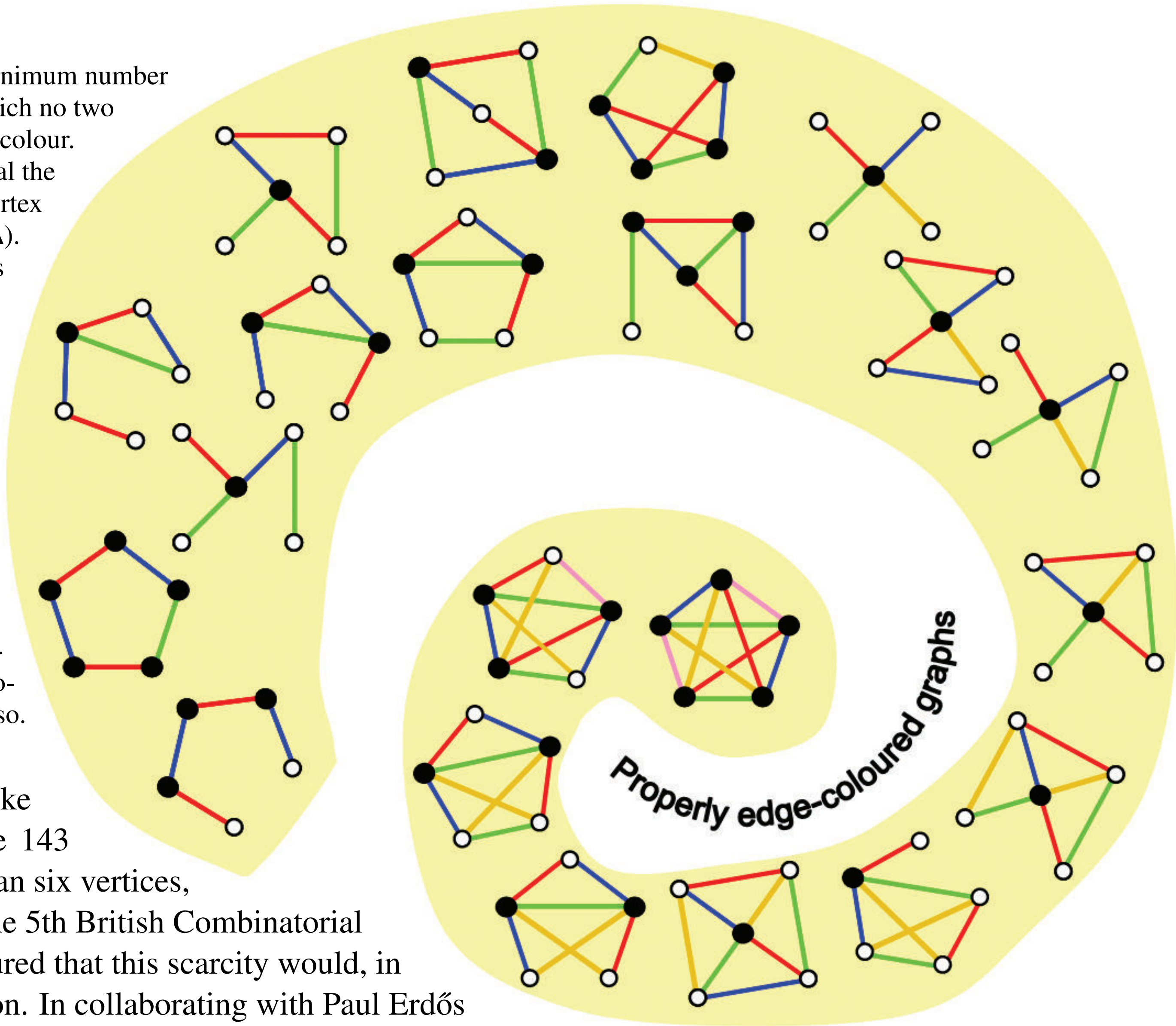
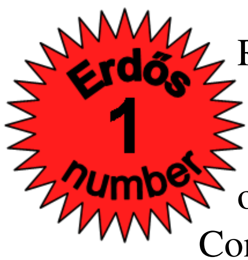
# THEOREM OF THE DAY

## **A Theorem of Erdős and Wilson on Edge Colouring** *Almost all graphs have edge-chromatic number equal to their maximum degree.*

The *edge-chromatic number* of a graph is the minimum number of colours sufficient for an edge colouring in which no two edges incident with any vertex receive the same colour. This minimum number must clearly at least equal the maximum number of edges incident with any vertex (the maximum *vertex degree*, usually denoted $\Delta$). A classic 1964 theorem of Vadim G. Vizing says that, on the other hand, it need never exceed $\Delta$ by more than one. Graphs with edge-chromatic number $\Delta + 1$ are called *class two* and Vizing also discovered that such graphs must have at least three vertices of maximum degree. In the graphs shown on the right, the vertices of degree $\Delta$ are drawn as black circles. All the twenty-one connected graphs on five vertices are shown; of these almost half have just one vertex of maximum degree. Erdős and Wilson proved that, as $n \to \infty$, the proportion of $n$-vertex graphs having more than one vertex of maximum degree tends to zero. It follows that the proportion of class two graphs must tend to zero also.




Robin J. Wilson and Lowell Beineke had observed in 1973 that, of the 143 connected graphs on not more than six vertices, only eight were of class two. At the 5th British Combinatorial Conference in 1975, Wilson conjectured that this scarcity would, in the limit, become an infinitesimal proportion. In collaborating with Paul Erdős to prove it, he joined one of the world's most exclusive clubs — those with Erdős number 1.

**Web link:** find the whole astonishing Erdős output here: www.renyi.hu/~p_erdos; Erdős–Wilson is 1977-20.

**Further reading:** *Graph Applications: an Introductory Approach* by Joan Aldous and Robin J. Wilson, Springer-Verlag London Ltd, 2003.